Math 1111 — Review Test 1

Name_____ Id_____ Section_____

Show your results clearly in order to get possible credits or partial credits. 10 points each.

1.Find the solution of each equation or system if any.

i) $5x - 6 = 12 - 10x$

ii)

$$\begin{cases} 3x - 2y &= -19 \\ x + 4y &= -4 \end{cases}$$

iii) $2x^3 - x^2 - x - 3 = 0$, given that $x = \frac{3}{2}$ as one of its roots.

2. i) Is $(3, -2)$ a solution of the following system ?

$$\begin{cases} \frac{2}{7}x - \frac{1}{5}y &= \frac{44}{35} \\ \frac{1}{3}x - \frac{5}{4}y &= \frac{7}{2}. \end{cases}$$

ii) The point of intersection of the lines $-7x + 3y = 4$ and $4x - 2y = 3$ is (*choose one*)

$$a)\ (\frac{17}{2}, \frac{29}{2}) \quad b)\ (-\frac{17}{2}, -\frac{37}{2}) \quad c)\ (-\frac{17}{2}, \frac{29}{2}) \quad d)\ (\frac{1}{2}, \frac{5}{2}).$$

3. Find the domain and range of each function.

(i) $y = \dfrac{1}{x-1}$ (ii) $y = \sqrt{-5x+1}$

(iii) $y = -|x|$ (iv) $y = [x]$

(v) $y = \dfrac{1}{\sqrt{1-x}}$ (vi)$y = \sqrt{\dfrac{t}{t+1}}$

vii) $y = |x| - \sqrt{-x}$

4. Let $H(x) = 1 - 2x^2$. Find the following:

(i)$H(0)$ (ii)$H(\sqrt{2})$ (iii) $H(\frac{5}{6})$

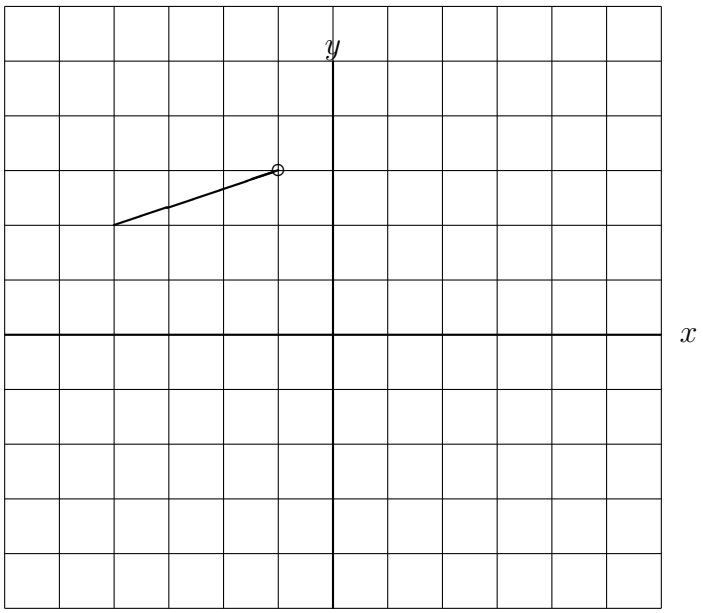
(iv) $H(x+h)$ (v) $\frac{H(x+h)-H(x)}{h}$

5. Compute and *simplify*

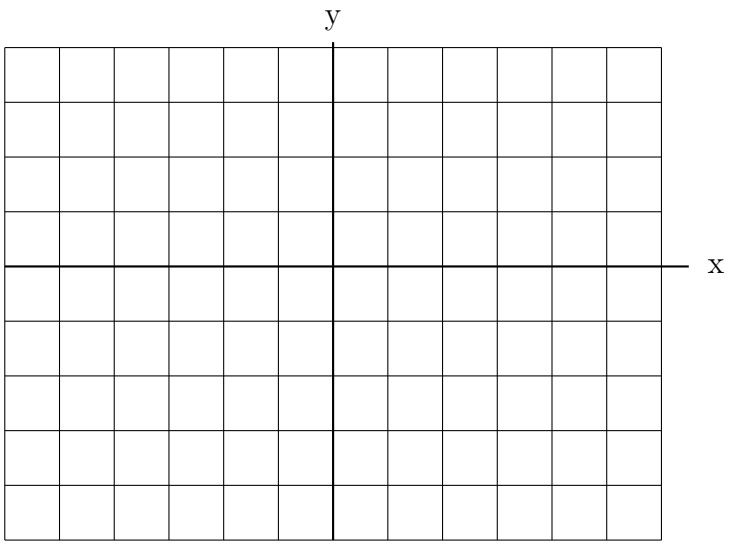
i) $f(2)$ and the difference quotient $\dfrac{f(x) - f(2)}{x - 2}$ for the function $f(x) = \dfrac{1}{1-x}$

ii) $\dfrac{g(x+h) - g(x)}{h}$ $(h \neq 0)$ for $g(x) = \sqrt{x+1}$.

6. Specify the domain and the range of the function whose graph is given (The axes are marked off in one-unit intervals).



7. Graph the function. Specify its vertex, all of its intercepts, the domain and the range.
   i) $y(x) = (x-1)^2 - 4$ ii) $p(x) = x^2 - 4x + 1$.



8. Let $f(x) = \dfrac{2}{x+1}$ and $g(x) = x^2$. Compute
   (i) $f \cdot g$ (ii)f/g (iii*) f^{-1}

9. Graph the functions. a)

$$h(x) = [x + 2] - 2$$

b)

$$g(x) = \begin{cases} |x| & if\ x < 1 \\ -3x + 4 & if\ x \geq 1 \end{cases}$$

c*) The graph of the function $y = f(x)$ is the line segment joining points (-2, -2) and (2, 1). Sketch and label the graphs of the following functions:

(a) $y = f(-x)$ (b) $y = -f(x)$ (c) $y = f(x + 3)$ (d) $y = f(x) + 1$

10*. Analyze the function algebraically: List its vertical asymptotes, horizontal asymptote, x-intercepts and y-intercept(if any). Then sketch a complete graph of the function.

$$y = \frac{400 - x^2}{(x - 100)^2}$$

11. Which point is farther away from the origin: $A(-1, 4)$ or $B(3, -2)$?

12. Is the triangle with vertices the origin O, $A(2, 4)$ and $B(10, 0)$ a right triangle?

13. The coordinates of A and B are $(-1, 2)$ and $(5, -3)$, respectively. If B is the midpoint of line segment AC, what are the coordinates of C?

14. Plot the point $(7, -8)$. Does this point lie on the graph of $y^2 = 9x + 1$?

15. Determine whether the graph of the function is symmetric with respect to
    (a) the origin
    (b) the x-axis
    (c) the y-axis.

i) $y^2 = 9x + 1$ ii) $H(x) = -\frac{3}{x^2} + \frac{x^4}{9} + 8$

16. Determine the center and radius of the circle $x^2 + y^2 - 8x + 6y - 24 = 0$.

17. A line passes through points $A(-1, 4)$ and $B(3, -2)$. Compute its slope and write out its equation.

18. Give the equation of the line with slope $m = -\frac{2}{3}$ and y-intercept $b = -\frac{5}{3}$.

19. Are the two lines $2x + 3y = -5$ and $3x - 2y = 5$ parallel or perpendicular or neither?

20. Find the equations of the tangent and normal lines of the circle $x^2 + y^2 = 25$ passing through $(3, -4)$. Write the answer in the form $y = mx + b$.

21. Jimmy buys a new car for \$21,000. After 10 years, the car has a salvage value of 2,000. Assuming linear depreciation, find the formula for the value V of the car after t years $(0 \leq t \leq 10)$.

22. (a) The perimeter of a rectangle is $16cm$. Express the area of the rectangle in terms of the width x.
    (b) The area of a rectangle is 75 cm^2. Express the perimeter as a function of the width x.

23. A baseball is thrown straight up, and its height as a function of time is given by $h = 32t - 32t^2$ (here h is in feet and t is in seconds).
(a) Find the height of the ball when $t = \frac{1}{2}$ and when $t = 1$.
(b) Find the maximum height of the ball and the time at which that height is attained.
(c) At what time the ball falls to the ground?

24*. Five hundred feet of fencing are available to enclose a rectangular pasture alongside a river, which serves as one side of the rectangle (so only three sides require fencing). Find the dimensions yielding the greatest area.

25. Simplify the expressions. a) $8i - (7 + 4i)$ b) $(5 - 4i)^2$
c) $\frac{1+3i}{2-i}$ d) $\frac{-8+\sqrt{-16}}{24}$ e) $\sqrt{-12}(\sqrt{-4} - \sqrt{2})$

26. Solve the following equations. Check your answer if necessary.
a) $x^2 - 12x + 27 = 0$ b) $2x^2 + 5x = 3$
c) $\frac{x-1}{4} + \frac{5}{x+1} = 2$ d) $3x^4 = 48x^2$
e) $\sqrt{2x - 3} + x = 3$ f) $\sqrt{x - 4} + \sqrt{x + 1} = 5$
g) $(x - 7)^{2/3} = 25$ h) $2x^{2/5} + 7x^{1/5} - 15 = 0$
i) $|2x - 5| = 11$

Answers to Test 1

3. (i) $x - 1 \neq 0$, so $x \neq 1$; Domain: $(-\infty, 1) \cup (1, \infty)$ Range: all real numbers except 0

(ii) Solve $-5x + 1 \geq 0$. Domain: $x \leq 1/5$. Range: $y \geq 0$

7 (i). x-int's: 3 and -1; y-int: -3; domain: $(-\infty, +\infty)$; range: $[-4, \infty)$; $y_{min} = -4$; y_{max}: none; it is decreasing on $(-\infty, 1]$, increasing on $[1, +\infty)$; vertex: $(1, -4)$

8. (a) $f \cdot g = \dfrac{2x^2}{x+1}$; (b) $f/g = \dfrac{2}{x^2(x+1)}$

11. $|OA| = \sqrt{17} > \sqrt{13} = |OB|$; so A is farther away from the Origin.

12. $|AB| = \sqrt{80}, |OA| = \sqrt{20}, |OB| = \sqrt{100}$; so $|OA|^2 + |AB|^2 = |OB|^2$ and it is a right triangle. Alternatively, we can use the slopes to show $OA \perp AB$, that is, compute $m_{OA} \cdot m_{AB} = -1$

13. $C(7, -8)$.

14. Yes.

15. (i) x-int: $-\dfrac{1}{9}$; y-int's: ± 1; the graph is symmetric in the x-axis.

16. $C(4, -3)$; $r = 7$.

17. $m = -\dfrac{3}{2}$; equation: $3x + 2y - 5 = 0$ or $y = -\dfrac{3}{2}x + \dfrac{5}{2}$.

18. $y = -\dfrac{2}{3}x - \dfrac{5}{3}$ or $2x + 3y + 5 = 0$.

19. $m_2 = -\dfrac{3}{2}$; so these two lines are neither parallel nor perpendicular.

21. $V = 20,000 - 1,900t$.