Math 1111 — Review Test 2

Name______________________________ Id______________________ Section_______

Show your results clearly in order to get possible credits or partial credits. 10 points each.

1.Find a) $(f \circ g)(x)$ b) $(g \circ f)(x)$ c) $(f \circ g)(2)$ d) domain of $f \circ g$

e) domain of $g \circ f$ for the following pair of functions.

i) $f(x) = 1/x$, $g(x) = 1/x$ (EX 2.6, Prob. 63)

ii) $f(x) = x^2 + 4$, $g(x) = \sqrt{1-x}$ (EX 2.6, Prob. 71).

iii) $f(x) = x^3 - 27$, $g(x) = \sqrt[3]{x+27}$

2. Solve polynomial equations.

i) $-x^4 + x^2 = 0$ (EX 3.2 Prob. 15)

ii) $-x^3 - x^2 + 5x - 3 = 0$ (EX 3.2 Prob 18)

iii) $2x^3 - x^2 - x - 3 = 0$.

3. a) Analyze the function algebraically: List its vertical asymptotes, holes, and horizontal asymptote. Then sketch a complete graph of the function.

$$y = \frac{x^2 - 1}{x^3 - 2x^2 + x}$$

b) Do the same as in a) for the rational function

$$y = \frac{2x^3 - 2x}{x^3 - 2x^2 + x}$$

4. a) Verify that the given pair of functions are inverse functions: $g(x) = \sqrt{x}; h(x) = x^2$ (Assume that the domain of both g and h is $[0, \infty)$).

b) Let $f(x) = 3x - 1$.

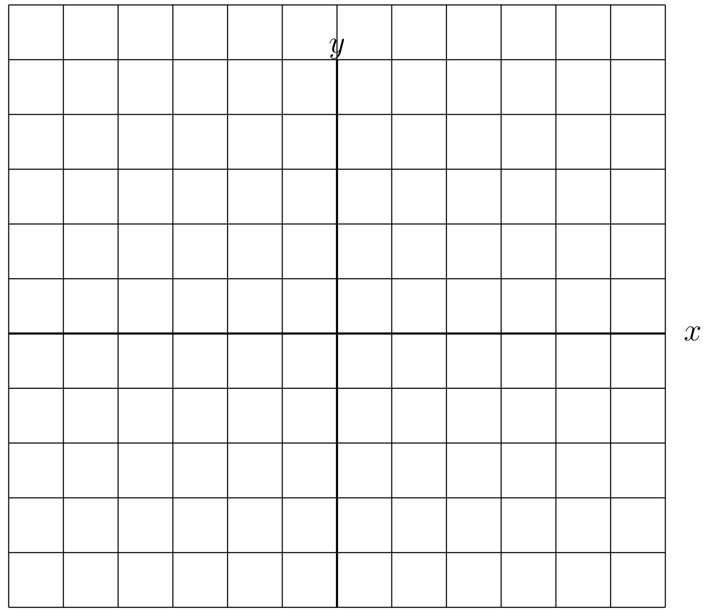
(i) Compute $f^{-1}(x)$.

(ii)Verify that $f(f^{-1}(x)) = x$ and that $f^{-1}(f(x)) = x$.

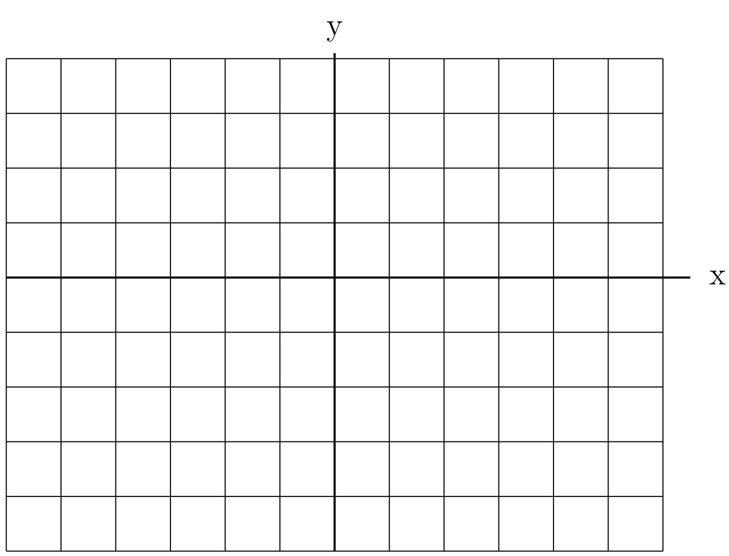
(iii)On the same set of axes, sketch the graph of f, f^{-1}, and the line $y = x$ (Note that the graphs of f and f^{-1} are symmetric about the line $y = x$).

5.Using the graph of the function $f(x) = \sqrt{x}$ $(x \geq 0)$ sketch and label the graphs of the following functions:

(a) $y = f(-x)$ (b) $y = -2f(x)$ (c) $y = f(x+3)$ (d) $y = f(x) + 1$
(e) $y = f(2x)$ (f) $y = 3f(2x-4) - 1$



6. Graph the function. Specify its vertex, x- and y- intercepts, horizontal or vertical asymptotes, if any, and the domain and the range.
   i) $y(x) = -3(x-1)^2 - 5$ ii) $p(x) = x^2 - 6x + 10$ iii) $y = \dfrac{400 - x^2}{(x-100)^2}$



(Hint: If necessary plot additional points between the x-intercepts and vertical asymptotes)

7. Find the inverse of a) $f(x) = \dfrac{2}{x+1}$, $x > -1$. b) $g(x) = (x-5)^2$, $x \leq 5$
   Explain why they have inverses (for example, increasing or decreasing property, horizontal line test, one to one correspondence)

8. Divide using synthetic or long division.

a) $(x^2 - 6x - 6x^3 + x^4) \div (6 + x)$ (EX 3.3, Prob. 26)

b) $(12x^2 + x - 4) \div (3x - 2)$ (EX 3.3, Prob. 7)

c) $(18x^4 + 9x^3 + 3x^2)/(3x^2 + 1)$ (EX 3.3, Prob. 15)

9. a) Sketch the graph of the polynomial $y = x^3(x - 1)$ by first specifying all intercepts. What the graph looks like when $|x|$ is large and when x is near its x-intercepts?

b) Graph the rationals $y = \dfrac{-2}{x+1}$ and $y = \dfrac{2x}{x+1}$. Specify their intercepts and asymptotes.

c) Graph the rationals $y = \dfrac{1}{(x-1)^2}$ and $y = \dfrac{1}{(x-1)^3}$. Specify their intercepts and asymptotes.

10. a) State the Fundamental Theorem of Algebra (see Section 3.4)

b) State the Linear Factorization Theorem

c) If a polynomial equation is of degree n, then counting multiple roots separately, the equation has ____ roots.

d) If $a + bi$ is a root of a polynomial equation with **real** coefficients ($b \neq 0$) then the complex conjugate ____ is also a root.

e) Using properties of polynomial equations solve $x^4 - 6x^3 + 22x^2 - 30x + 13 = 0$ (Section 3.4, Check point 5)

11* (optional) (a) The intensity of light received at a source varies inversely as the square of the distance from the source. A particular light has an intensity of 20 foot-candles at 15 feet. What is the light's intensity at 10 feet.

12. A ball is thrown vertically upward from the top of the Leaning Tower of Pisa (190 feet high) with an initial velocity of 96 feet/sec. During which time period will the ball's height exceed that of the tower?

(Hint: This is a word problem involving solving a quadratic function inequality. The formula for free falling object is given by

$$h(t) = -16t^2 + v_0 t + h_0$$

where $h = h(t)$ is the height of the ball off the ground level at time t, v_0 the initial velocity, h_0 the initial height of the ball)

13. A field bordering a straight stream is to be enclosed. The side bordering the stream is not to be fenced. If 1000 yards of fencing material is to be used, what are the dimensions of the largest rectangular field that can be fenced? What is the maximum area? (Review Ex. Prob. 9)