Math 1111 — Review Exam

Name____________________ Id____________________ Section________

Show all your work and simplify your answers as much as possible to receive credits or partial credits.

1. Specify the domain for each of the following functions:
(a) $f(x) = \dfrac{1}{1-x}$ (b) $g(x) = \sqrt{1-x}$ (c) $h(x) = \dfrac{1}{\sqrt{1-x}}$
2. Determine the center and the radius of the circle $25x^2 - 30x + 25y^2 + 50y - 30 = 0$.
3. Find the equation of the line a) perpendicular to the line $-3x + 9y - 1 = 0$ and passing through the point $(-1, -2)$.
b) passing through points $A(-1, 4)$ and $B(3, -2)$.
c) having slope $m = -\dfrac{2}{3}$ and y-intercept $b = -\dfrac{5}{3}$.
d) Are the two lines $2x + 3y = -5$ and $3x - 2y = 5$ parallel or perpendicular or neither?
e) (bonus) Find the equation of the tangent line of the circle $x^2 + y^2 = 25$ passing through $(3, -4)$. Write the answer in the form $y = mx + b$.
4. Let $f(x) = \frac{x+4}{x-4}$ and $g(x) = -x + 3$. Compute
(a)the composition $f \circ g$
(b)the inverse function f^{-1} of f.
c) Given $f(x) = \dfrac{1}{x+1}$ and $g(x) = x^2$. Please compute:
(i) $f \cdot g$ (ii) $f \circ g$ (iii) $g \circ f$ (iv) f^{-1}
5. Determine whether the graph of the function $H(x) = -\frac{3}{x^2} + \frac{x^4}{9} + 8$ is symmetric with respect to
(a) the origin
(b) the x-axis
(c) the y-axis. d) Test all three type of symmetry for $y^2 = 9x + 1$.
6.a) Graph the function $p(x) = x^2 - 4x + 1$. Specify the vertex, axis of symmetry, minimum/maximum value and intercepts.
b) Graph the function $y = (x-1)^2 - 4$. Specify all of its intercepts, the domain, the range, the maximum value and minimum value, the interval where the function is rising, the interval where the function is falling and the turning point. Does it possess any inverse? Why?
7. Solve the equations a) $10^{x^3} = 40$.
b) $\log_4 x + \log_4(6-x) - \frac{3}{2} = 0$. Check your answer.
c) $(10^x)^2 = 40$. Express your solution in natural logarithm.
d) $\log_3 x + \log_3(x+2) = 1$.
e) $2e^{2x-1} = 4$. Express your answer in natural logarithm.
f) $\log_2(x+3) = 2 + \log_2 x$.
g) $4\log_3 x = 6 - \log_3 64$
8. Evaluate *without using a calculator*: a) $2\log_3 5 - \log_3 75$.
b) $\log_2 e + \log_3 e$ in terms of natural logarithm. c) $\log_3 144 - 2\log_3 4$.

9*. If $E(x) = e^x$, show that

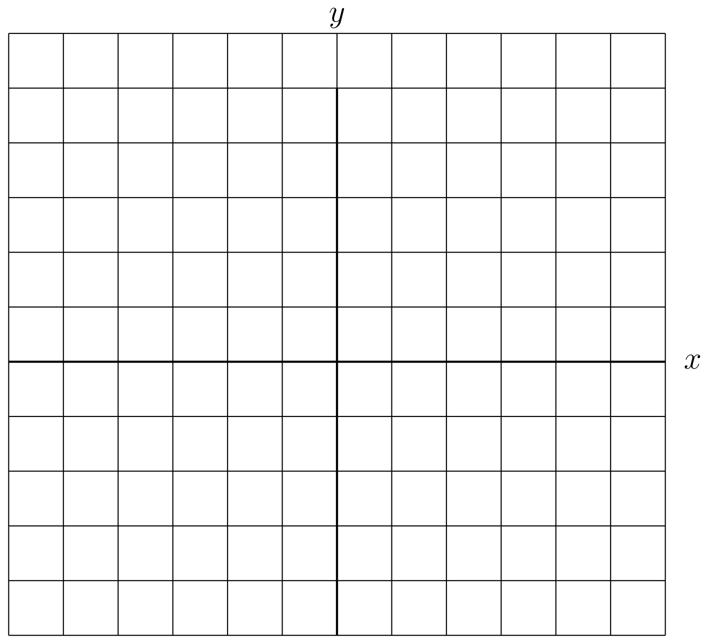
$$\frac{E(x+h) - E(x)}{h} = e^x(\frac{e^h - 1}{h}).$$

10. Graph the following functions. In each case, specify the domain, range, intercepts and horizontal/vertical asymptote. a) $y = e^{-x}$ and $y = \ln(x+1)$.
b) $y = e^{x+2} + 2$ and $y = \ln(x-2) - 2$. Verify that the pair in b) are inverse functions.
11. Solve the equation a) $3x + 13 = -5x - 4$. b) $x^2 - 12x - 11 = 0$. c) $2x^3 - x^2 - x - 3 = 0$, given that $x = \frac{3}{2}$ as one of its roots.
12. The point of intersection of the lines $-7x + 3y = 4$ and $4x - 2y = 3$ is (*choose one*)

$$a)\ (\frac{17}{2}, \frac{29}{2}) \quad b)\ (-\frac{17}{2}, -\frac{37}{2}) \quad c)\ (-\frac{17}{2}, \frac{29}{2}) \quad d)\ (\frac{1}{2}, \frac{5}{2}).$$

13. Five hundred feet of fencing are available to enclose a rectangular pasture alongside a river, which serves as one side of the rectangle (so only three sides require fencing). Find the dimensions yielding the greatest area.
14. Tom invests \$7000 at 5% per year, compounded continuously.
(a) How much is in the account after 7 years?
(b) When will the balance reach \$14000? (c) What if the interest is compounded monthly? Give answer to the same questions in a), b).
15. Suppose the population of the United States at the year 1960 was 115 million and at the year 1990 was 230 million. Assuming the growth law $P = P_0 e^{rt}$, what is the population at a) 1996 b) present time (2007)?
16. a) The intermediate value property for polynomials tells: Let $y = f(x)$ be a polynomial on [a,b]. If $f(a) > 0$ and $f(b) < 0$, then ________
b) The rational zero theorem for a polynomial: Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ have integer coefficients a_i, $i = 0, 1, \ldots, n$. If p/q is a reduced zero of $f(x)$, then ________
c) The remainder theorem is: Given a polynomial $f(x)$ as a dividend and $d(x)$ as a divisor, then ________. Hence if $x = r$ is a root of $f(x)$ then ________ is a factor of $f(x)$.
d) State the Fundamental Theorem of Algebra (see Section 3.4)
and the Linear Factorization Theorem. Give an example to verify your statements.
e) The change of base formula for logarithm function is: for $b, x > 0$ $\log_b x =$
17. Plot the point $(7, -8)$. Does this point lie on the graph of $y^2 = 9x + 1$?
18. Divide using synthetic or long division. (Show your work)
a) $(x^2 - 6x - 6x^3 + x^4) \div (6 + x)$
b) $(x^4 + 4x^3 + 6x^2 + 4x + 1) \div (x^2 - 1)$
19* (optional) Analyze the function algebraically: List its vertical asymptotes, holes, and horizontal asymptote. Then sketch a complete graph of the function.
a) $y = (x-3)^2(x+2)$
b) $y = \dfrac{1}{2x+2}$

$$c^*)\quad y = \frac{x^2 - 1}{x^3 - 2x^2 + x} \qquad\qquad d^*)\quad y = \frac{2x^3 - 2x}{x^3 - 2x^2 + x}$$



20. The perimeter of a rectangle is $16cm$. Express the area of the rectangle in terms of the width x.

Answers To Review Exam

1. (a) $1-x\neq 0$, so $x\neq 1$; (b) $1-x\geq 0$, so $x\leq 1$; (c) $1-x\geq 0$ and $1-x\neq 0$, so $x<1$.

2. Complete the square: $25(x-\frac{3}{5})^2+25(y+1)^2=64$
Divided by 25 on both sides: $(x-\frac{3}{5})^2+(y+1)^2=\frac{64}{25}$
Ans: center $(\frac{3}{5},-1)$, radius $\frac{8}{5}$.

3. a) Slope of the line $-3x+9y-1=0$ is $\frac{1}{3}\Longrightarrow$
slope of the desired line is -3.
By the point-slope formula, the equation of the desired line is $y+2=-3(x+1)$ or $3x+y+5=0$.

4. a)

$$\frac{x-7}{x+1};$$

b)

$$f^{-1}(x)=4\frac{x+1}{x-1}.$$

c) (i) $f\cdot g=\dfrac{x^2}{x+1}$;
(ii) $f\circ g=f(g(x))=f(x^2)=\dfrac{1}{x^2+1}$;
(iii) $g\circ f=g(f(x))=g(\dfrac{1}{x+1})=(\dfrac{1}{x+1})^2=\dfrac{1}{(x+1)^2}$;
(iv) Fuction f is $y=\dfrac{1}{x+1}$, in which we interchange x and y to get

$$x=\frac{1}{y+1},\quad \text{so}\quad x(y+1)=1,\quad \text{i.e.}\quad xy+x=1,\quad \text{i.e.}\quad xy=1-x$$

$$\text{so}\qquad f^{-1}:\quad y=\frac{1-x}{x}=\frac{1}{x}-1$$

5. Only symmetric in the y-axis.

6 a) Vertex $(2,-3)$; $p_{min}=-3$; axis of symmetry $x=2$; y-int: 1; x-int: $2+\sqrt{3}$.

b) x-int's: -1 and 3; y-int: -3; domain: $(-\infty,+\infty)$; range: $[-4,\infty)$; $y_{min}=-4$; y_{max}: none; it is falling on $(-\infty,1]$, rising on $[1,+\infty)$; turning point: $(1,-4)$; it has no inverse since it fails the Horizontal Line Test.

7. a) Take natural logarithm on both sides:

$$\begin{aligned} x^3 \ln 10 &= \ln 40 \\ x^3 &= \tfrac{\ln 40}{\ln 10} \\ Ans: x &= (\tfrac{\ln 40}{\ln 10})^{\frac{1}{3}} \end{aligned}$$

b) $x = 2, 4$ Hint: Use product rule and the definition of logarithm function

c) $x = \dfrac{1 + \ln 2}{2}$

d) $x = 1$, ($x = -4$ is an extra solution)

9. Proof.

$$Left = \frac{e^{x+h} - e^x}{h} = \frac{e^x e^h - e^x}{h} = \frac{e^x(e^h - 1)}{h} = Right.$$

10. a) Exp Domain: all real numbers; range: $y > 0$; no x-int; y-int: 1; h-asymptote: x-axis
Log Domain: $x > -1$; range: all real numbers; x-int: 0; y-int: 0; v-asymptote: $x = -1$
(b) Exp Domain: all real numbers; range: $y > 2$; no x-int; y-int: $e^2 + 2$; h-asymptote: $y = 2$
Log Domain: $x > 2$; range: all real numbers; x-int: $e^2 + 2$; y-int: none; v-asymptote: $x = 2$.

11.

$$x = \frac{3}{2}, \quad \frac{-1 + \sqrt{3}i}{2}$$

12. (b)

13. 125 by 250 ft^2

14. (a) $7000 \times e^{0.05 \cdot 7} = 7000e^{0.35} \approx \$9933.47$
(b)

$$14000 = 7000 \times e^{0.05t}$$

Solving the equation yields

$$\begin{aligned} \ln 2 &= .05t \\ t &= \tfrac{\ln 2}{.05} \approx 14yrs. \end{aligned}$$