Chapter 1. Differentiation.
1.1 Limits: A Numerical and Graphical Approach
1.2 Algebraic Limits and Continuity

- [Review section (continued)] *Power Functions with Rational Exponents*

(32) **Definition.** Let m and n be positive integers. Then

$$a^{m/n} = \sqrt[n]{a^m}, \qquad a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{\sqrt[n]{a^m}}.$$

(33) Rewrite $\sqrt[3]{x}$, $\sqrt{x^3}$, and $\dfrac{1}{\sqrt[4]{r^5}}$ as an equivalent expression with rational exponents.

(34) Simplify $8^{2/3}$ and $9^{3/2}$.

- *Supply and Demand Functions**

(35) * Let x be the unit price of some product. Let $q = D(x)$ be a decreasing function that models the *demand* and $q = S(x)$ be an increasing function that models the *supply*. The point of intersection of the two curves, denoted (X_E, q_E), is called the **equilibrium point**.

(36) * Find the equilibrium point for each pair of demand and supply functions.
(a) Demand: $q = D(x) = 3 - x$; Supply: $q = S(x) = \sqrt{2x+2}$
(b) Demand: $q = D(x) = 4/x$; Supply: $q = S(x) = x/4$
(c) Demand: $q = D(x) = 8800 - 30x$; Supply: $q = S(x) = 7000 + 15x$

- **Limit of A Function**

(37) Let $f(x) = x^2 + 1$. We will find out what value does $f(x)$ approach as x approaches 1.
(a) First let x approach 1 from the left. Compute $f(0.9)$, $f(0.99)$, $f(0.999)$, and $f(0.9999)$.
(b) Next let x approach 1 from the right. Compute $f(1.1)$, $f(1.01)$, $f(1.001)$, and $f(1.0001)$.

(38) Redo Problem 37 with function $f(x) = \begin{cases} x+2, & \text{for } x \geq 1, \\ \sqrt{3+x}, & \text{for } x < 1. \end{cases}$

(39) **Definition.** As x approaches a (from both sides), the **limit** of $f(x)$ is L, written $\lim\limits_{x\to a} f(x) = L$, if all values of $f(x)$ are close to L for values of x that are sufficiently close, but not necessarily equal, to a. The limit L, if it exists, must be a unique real number.

We write $\lim\limits_{x\to a^-} f(x)$ to indicate the limit from the left (i.e. $x < a$), and $\lim\limits_{x\to a^+} f(x)$ to indicate the limit from the right (i.e. $x > a$), if we want to specify the side from which x-values approach a. These are called **left-hand limits** and **right-hand limits**, respectively.

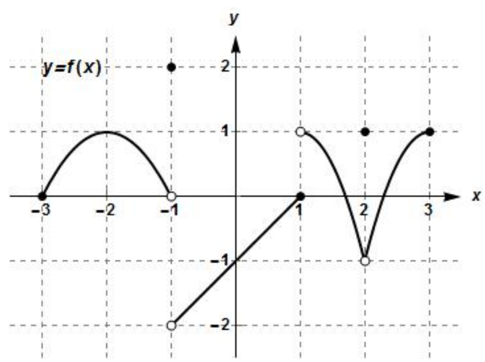
(40) **Theorem.** *As x approaches a, the limit of $f(x)$ is L if and only if the left-hand and right-hand limits exist and are equal to L.*

(41) Let $f(x) = \dfrac{x^2 - 4}{x - 2}$.

(a) Does $f(2)$ exist?
(b) Compute $f(1.9)$, $f(1.99)$, $f(1.999)$, and $f(1.9999)$.
(c) Compute $f(2.1)$, $f(2.01)$, $f(2.001)$, and $f(2.0001)$.
(d) What is $\lim_{x\to 2} f(x)$?

(42) Let $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & \text{for } x \neq 2, \\ -1, & \text{for } x = 2. \end{cases}$ Find $f(2)$ and $\lim_{x\to 2} f(x)$.

(43) The graph of the function $y = f(x)$ on $-3 \le x \le 3$ is given as below.



Find the values of $f(-1)$, $f(0)$, $f(1)$, $f(2)$, $\lim_{x\to -1^-} f(x)$, $\lim_{x\to 0} f(x)$, $\lim_{x\to 1^+} f(x)$, and $\lim_{x\to 2} f(x)$, if they exist.

(44) Let $f(x) = \dfrac{1}{(x-1)^3}$.

(a) First let x approach 1 from the left. Compute $f(0.9)$, $f(0.99)$, $f(0.999)$, and $f(0.9999)$.
(b) Next let x approach 1 from the right. Compute $f(1.1)$, $f(1.01)$, $f(1.001)$, and $f(1.0001)$.

(45) Let $f(x) = \dfrac{x}{x^2 + 1}$.

(a) First let x approach ∞. Compute $f(100)$, $f(1000)$, $f(10000)$, and $f(100000)$.
(b) Next let x approach $-\infty$. Compute $f(-100)$, $f(-1000)$, $f(-10000)$, and $f(-100000)$.

Videos on Chapter 1:Differentiation from MLM Plus