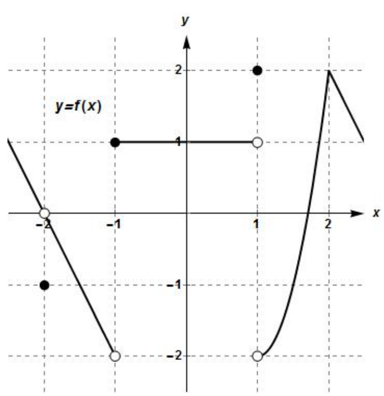
Chapter 1. Differentiation.
1.3* Average Rates of Change
1.4 Differentiation Using Limits and Difference Quotients
1.5 Leibniz Notation and The Power and Sum-Difference Rules (Part I. The Power Rule)

- *Algebraic Limits*
  (46) Find the limits.
  (a) $\lim_{x \to -1} (2x^2 - 4x + 4)$
  (b) $\lim_{x \to -3} \dfrac{x^2 + 2x + 2}{x - x^3}$
  (c) $\lim_{x \to -3} \sqrt{\dfrac{x^2 + 2x + 2}{x - x^3}}$
  (47) Find the limits.
  (a) $\lim_{x \to -1} \dfrac{x^2 - x - 2}{x^2 - 1}$
  (b) $\lim_{x \to -3} \dfrac{x + 3}{x^2 + 4x + 3}$
  (c) $\lim_{x \to 4} \dfrac{x^2 - 3x - 4}{x^2 - 5x + 4}$

- *Continuity*
  (48) **Definition.** A function f is **continuous** at $x = a$ if: (a) $f(a)$ exists, (b) $\lim_{x \to a} f(x)$ exists, and (c) $\lim_{x \to a} f(x) = f(a)$.
  A function is **continuous over an interval *I*** if it is continuous at each point a in I. If f is not continuous at $x = a$, we say that f is **discontinuous**, or has a **discontinuity**, at $x = a$.
  (49) The graph of the function $y = f(x)$ is given as follows. At what points of x is f **not** continuous?



  (50) Determine whether the function is continuous at the given point.

(a) $f(x) = x^3 + 2x, \quad \text{at } x = 2$

(b) $f(x) = \begin{cases} 2x - 3, & \text{for } x \geq 3, \\ 12 - x^2, & \text{for } x < 3, \end{cases} \quad \text{at } x = 3$

(c) $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & \text{for } x \neq 1, \\ 2, & \text{for } x = 1, \end{cases} \quad \text{at } x = 1$

- *Difference Quotient and Derivatives*

(51) The following table shows total production of suits at a company during one morning work. What was the average number of suits produced per hour from 9 am to 11 am?

Time (number of hours since 8 am)	0	1	2	3	4
Total number of suits produced	0	20	55	64	100

(52) **Definition.** The average rate of change of $f(x)$ with respect to x is also called the **difference quotient**. It is given by

$$\frac{\Delta y}{\Delta x} = \frac{\Delta f(x)}{\Delta x} := \frac{f(x+h) - f(x)}{h}, \qquad \text{where } h \neq 0.$$

(53) For $f(x) = x^2$, find the difference quotient for $x = 1$ and
(a) $h = 1$, $h = 0.5$, $h = 0.2$, and $h = 0.1$ (h approaches 0 from the right)
(b) $h = -1$, $h = -0.5$, $h = -0.2$, and $h = -0.1$ (h approaches 0 from the left)

(54) **Definition.** For a function $y = f(x)$, its **derivative** at x is the function f' (also written as $\dfrac{dy}{dx}$ or $\dfrac{df(x)}{dx}$) defined by

$$f'(x) = \frac{df(x)}{dx} := \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists. If $f'(x)$ exists, then we say that f is **differentiable** at x. The process of finding a derivative is called **differentiation**.

Geometrically, the derivative of f at x is the slope of the tangent line at $(x, f(x))$. This limit is also called the **instantaneous rate of change** of f at x.

(55) For each function $f(x)$, first find $f'(x)$, then find $f'(-3)$ and $f'(2)$.
(a) $f(x) = 4x - 2$
(b) $f(x) = \dfrac{1}{x}$
(c) $f(x) = x^2$

Section 1.5. (Part I) The power rule.

Let n be a real number.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

for all x in the domain of the function $f(x) = x^n$

Constant multipler Rule:

$$\tfrac{d}{dx}\Big(cf(x)\Big) = c\tfrac{d}{dx}f(x)$$

$$\text{or} \qquad \Big(cf(x)\Big)' = cf'(x)$$

Ex. Find an equation of the tangent line to the graph of $f(x) = -2x^3$ at $(-1, 2)$
[Answer] The equation of the tangent line is $\boxed{y = -6x - 4}$

Solution. The slope of the tangent line is equal to the derivative at $x = -1$:

$$y'(-1) = f'(-1) = -6x^2|_{x=-1} = -6$$

By the slope-point form of the line equation we have

$$y - 2 = (-6)(x - (-1))$$

Simplify :

$$y = -6x - 4.$$

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Videos on Chapter 1:Differentiation from MLM Plus