- *Elasticity of Demand*

(84) A DVD rental company has found that demand for rentals of its DVDs is given by $q = D(x) = 120 - 20x$, where q is the number of DVDs rented per day at x dollars per rental. Suppose that the price per rental is currently \$2.
  (a) Find the demand per day.
  (b) If the price per rental is raised by 10%, then how much drop is it in demand?
  (c) How much does the total revenue change?
  (d) Now assume the current price per rental is \$4. Do (a)–(c) again.

(85) **Definition.** The **elasticity of demand** E is given as a function of price x by

$$E(x) = -\frac{x \cdot D'(x)}{D(x)}.$$

(86) Now let us revisit Problem 84.
  (a) Find the elasticity of demand E as a function of x. Then find the elasticity at $x = 2$ and at $x = 4$.
  (b) Find the value of x for which $E(x) = 1$.
  (c) Find the total revenue R as a function of x. Then find the price x at which the total revenue is a maximum.

(87) **Theorem.** *Total revenue is increasing at those x-values for which $E(x) < 1$, is decreasing at those x-values for which $E(x) > 1$, and is maximized at the value(s) of x for which $E(x) = 1$.*

(88) **Definition.** The demand is **inelastic** if $E(x) < 1$, has **unit elasticity** if $E(x) = 1$, and is **elastic** if $E(x) > 1$.

Chapter 2.
2.1 Exponential and Logarithmic Functions of the Natural Base e
2.2 Derivatives of Exponential (Base-e) Functions
2.3 Derivatives of Natural Logarithm Functions
2.4* Applications: Uninhibited and Limited Growth Models
2.5* Applications: Decay

- *Exponential Functions*

(89) **Definition.** An **exponential function** f is given by $f(x) = a_0 \cdot a^x$, where x can be any real number, a_0 is a real number, and $a > 0$ and $a \neq 1$. The number a is the **base**.

(90) Graph $y = 2^x$ and $y = (\frac{1}{2})^x$.

(91) **Definition.** We call $e = \lim_{h \to 0}(1 + h)^{1/h} \approx 2.718281828459$ the **natural base**.

(92) Differentiate the following functions.
  (a) $y = 5e^x$ (b) $y = x^4e^x$ (c) $y = \dfrac{e^x}{x^4}$ (d) $y = e^{-x}$ (e) $y = e^{-x^3+x-3}$

- *Logarithmic Functions*
  (93) **Definition.** For $a > 0$ and $a \neq 1$, a **logarithm** $y := \log_a x$ means $a^y = x$. The number a is called the **logarithmic base**. If $a = e$, then $\log_e x$ is called the **natural logarithm** of x (abbreviated $\ln x$).
  (94) Solve for x.
  (a) $\log_3 27 = x$ (b) $\log_x 64 = 3$ (c) $\log_6 x = -1$ (d) $e^{-0.25x} = 0.58$
  (95) **Theorem.** *The function* $\ln x$ *exists only for positive numbers* x. *The domain is* $(0, \infty)$. *When* $0 < x < 1$, $\ln x < 0$. *When* $x = 1$, $\ln x = 0$. *When* $x > 1$, $\ln x > 0$. *The function* $\ln x$ *is an increasing function. The range is the entire real line* $(-\infty, \infty)$.
  (96) **Theorem.** $\dfrac{d}{dx}\ln x = \dfrac{1}{x}, \quad \dfrac{d}{dx}\ln|x| = \dfrac{1}{x}.$
  (97) Differentiate the following functions.
  (a) $y = 8\ln x$ (b) $y = x^4 \ln x$ (c) $y = \dfrac{\ln x}{x^4 + 1}$ (d) $y = \ln(6x^2 - 3x)$
- *Exponential Growth and Decay*
  (98) **Theorem.** *A function* $P = P(t)$ *satisfies* $P' = kP$ *if and only if* $P = Ce^{kt}$ *for some constant* C. *If* $k > 0$, P *is said to grow exponentially. If* $k < 0$, P *is said to decay exponentially.*
  (99) Suppose P_0, in dollars, is invested in the Von Newmann Hi-Yield Fund, with interest compounded continuously at 7% per year. That is, the balance P grows at the rate $P' = 0.07P$.
  (a) Find the function that satisfies the equation.
  (b) Suppose that \$100 is invested. What is the balance after one year?
  (c) In what period of time will an investment of \$100 double itself?
  (100) A person wants to make an initial investment P_0 that will grow to \$100,000 in 10 years. Suppose the interest is compounded continuously at 4% per year. What should the initial investment be?

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