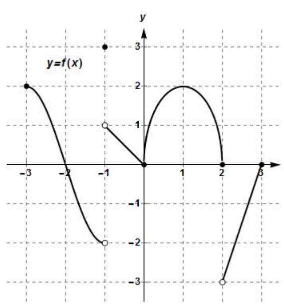
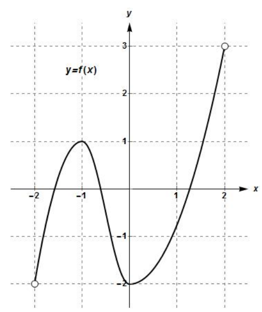
Chap.3 Applications of Differentiation
3.1 Using First Derivatives to Classify Maximum and Minimum Values and Sketch Graphs
3.2 Using Second Derivatives to Classify Maximum and Minimum Values and Sketch Graphs
3.3 Graph Sketching: Asymptotes and Rational Functions
3.4 Optimization: Finding Absolute Maximum and Minimum Values
3.6* Marginals, Differentials and Linearization
3.9* Related Rates

3.1. • *Maximum and Minimum Values*

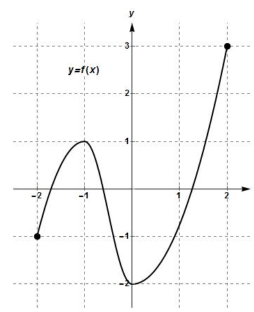
(67) The followings are graphs of three functions.



(1)



(2)



(3)

(a) Find the **open** intervals on which the function is increasing and decreasing.
(b) Identify the function's relative and absolute extreme values, if any, saying where they occur.

[Answer]
(a) Function $y = f(x)$ in Fig. (1).
(Open) Interval for increasing: $(0, 1)$, $(2, 3)$
(Open) Interval for decreasing: $(-3, -1)$, $(-1, 0)$, $(1, 2)$
(b) In Fig. (2),
(Open) Interval for increasing: $(-2, -1)$, $(0, 2)$
(Open) Interval for decreasing: $(-1, 0)$
(c) Fig. (3),
(Open) Interval for increasing: $(-2, -1)$, $(0, 2)$
(Open) Interval for decreasing: $(-1, 0)$

(68) **Definition.** A **critical value** of a function f is any number c in the domain of f for which the tangent line at $(c, f(c))$ is horizontal of for which the derivative does not exist. That is, c is a critical value if $f(c)$ exists and $f'(c) = 0$ or $f'(c)$ does not exist.

(69) **Theorem.** (a) *Let f be differentiable over an open interval I. If $f'(x) > 0$ for all $x \in I$, then f is increasing over I. If $f'(x) < 0$ for all $x \in I$, then f is decreasing over I.*
(b) *If a function f has a relative extreme value $f(c)$ on an open interval, then c is a critical value.*

- *Maximum and Minimum Values (Continued)*

(70) Given
(1) $f'(x) = (x-1)(x+2)^2$,
(2) $g(x) = x^4 - 4x^3 + 4x^2$,
(3) $h(x) = x^3 - 3x + 2$
answer the following questions.
(a) Find the critical points of each function.
(b) Find the **open** intervals on which the function is increasing and decreasing.
(c) At what point(s) of x will the function attain its relative extreme values? If possible, find out the relative extreme values.

(71) **Definition.** Suppose that f is a function whose derivative f' exists at every point in an open interval I. Then f is **concave up** on I if f' is increasing over I, and f is **concave down** on I if f' is decreasing over I. A point $(c, f(c))$ where the graph has a tangent line and where the concavity changes is a **point of inflection**.

(72) **Test for Concavity.** *If $f''(x) > 0$ on an interval I, then the graph of f is concave up on I. If $f''(x) < 0$ on an interval I, then the graph of f is concave down on I.*

(73) **The Second Derivative Test for Relative Extrema.** *Suppose that f is differentiable for every x in an open interval (a, b) and that there is a critical value c in (a, b) for which $f'(c) = 0$. Then* (a) *$f(c)$ is a relative minimum if $f''(c) > 0$, and* (b) *$f(c)$ is a relative maximum if $f''(c) < 0$.*
When $f''(c) = 0$, the test is inconclusive.

(74) Find all relative exterma and classify each as a maximum or minimum.
(a) $f(x) = 8x^3 - 6x + 1$
(b) $f(x) = 3x^5 - 20x^3$

- *Absolute Maxima and Minima*

(75) **Extreme Value Theorem.** *A continuous function f defined over a closed interval $[a, b]$ must have an absolute maximum value and an absolute minimum value over $[a, b]$.*

(76) Find the absolute maximum and minimum values of the following functions.
(a) $f(x) = x^3 - 3x \quad$ on $[-2, 2]$
(b) $f(x) = \dfrac{x^2}{x-2} \quad$ on $[3, 6]$
(c) $f(x) = (x-1)(x-3) \quad$ on $[0, 3]$

- *Maximum-Minimum Problems*

(77) Rudy wants to enclose a 100 square feet rectangular region to be used for a garden. He will use fencing which costs \$16 per foot along three sides, and fencing which costs \$34 per foot along the fourth side. Find the dimensions which give the minimum total cost, and find the minimum total cost.

(78) An open-top box is to be made by cutting small congruent squares from the corners of 6-in.-by-6-in. sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?

(79) The U.S. Postal Service will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 108 inches. What dimensions will give a box *with a square end* the largest possible volume?

(80) Cruzing Tunes determines that in order to sell x units of a new car stereo, the price per unit, in dollars, must be $p(x) = 1000 - x$. It also determines that the total cost of producing x units is given by $C(x) = 3000 + 20x$. What price per unit must be charged in order to make this maximum profit?

(81) By keeping records, a theater determines that at a ticket price of \$26, it averages 1000 people in attendance. For every drop in price of \$1, it gains 50 customers. Each customer spends an average of \$4 on concessions. What ticket price should the theater charge in order to maximize total revenue?

- *Marginals*

(82) **Definition.** Let $C(x)$, $R(x)$, and $P(x)$ represent, respectively, the total cost, revenue, and profit from the production and sale of x items.

  (a) The **marginal cost** at x, given by $C'(x)$, is the approximate cost of the $(x+1)$st item: $C'(x) \approx C(x+1) - C(x)$, or $C(x+1) \approx C(x) + C'(x)$.

  (b) The **marginal revenue** at x, given by $R'(x)$, is the approximate revenue from the $(x+1)$st item: $R'(x) \approx R(x+1) - R(x)$, or $R(x+1) \approx R(x) + R'(x)$.

  (c) The **marginal profit** at x, given by $P'(x)$, is the approximate profit from the $(x+1)$st item: $P'(x) \approx P(x+1) - P(x)$, or $P(x+1) \approx P(x) + P'(x)$.

(83) Given $R(x) = 5x$ and $C(x) = 0.001x^2 + 1.2x + 60$, find the following.

  (a) Total profit $P(x)$

  (b) Total cost, revenue, and profit from the production and sale of 100 units of the product

  (c) The marginal cost, revenue, and profit when 50 units are produced and sold

Videos on Chap.3 from MLM Plus