Chap. 4 Integration
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• §4.1 *Antidifferentiation*

(101) **Definition.** A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I. The process of recovering a function F from its derivative f is called **antidifferentiation**.

(102) Determine whether $F(x)$ is an antiderivative of $f(x) = e^{2x} + x$.
(a) $F(x) = e^{2x} + \frac{x^2}{2}$ (b) $F(x) = \frac{1}{2}(e^{2x} + x^2)$ (c) $F(x) = \frac{1}{2}(e^{2x} + x^2) + 2016$

(103) §4.1, Question # 21. Evaluate $\int (3x^{-1} + 6x^{-4})\, dx$.

Solution. Step 1. Split the integral into two parts $\int 3x^{-1} + \int 6x^{-4}$.
Step 2. Evaluate each of the two integrals (or anti-derivatives).
$\int 3x^{-1}dx = 3\int x^{-1}dx = 3\ln|x|$ (by the formula the anti-derivative of $1/x$ is $\ln|x| + C$). (logarithm rule for integration)
For the second part, $\int 6x^{-4}dx = 6\int x^{-4}dx$.
Since $\int x^{-4} = \frac{x^{-4+1}}{(-4+1)} + C = \frac{x^{-3}}{-3} + C$, (power rule for integration),
it follows that

$$\int 6x^{-4}dx = 6\int x^{-4}dx$$
$$=6\frac{x^{-3}}{-3} + C = (-2)x^{-3} + C = -\frac{2}{x^3} + C.$$

Step 3. Finally we combine the two above to obtain

$$\int (\frac{3}{x} + \frac{6}{x^4})dx = 3\ln|x| - \frac{2}{x^3} + C.$$

This completes the calculation. □

(104) Antidifferentiate the following functions.
(a) $f(x) = \frac{1}{\sqrt{x}} + 5x^3$
(b) $f(x) = e^{3x}$
(c) $f(x) = \frac{3x^4 - 2x^2 + x - 1}{x^2}$

(105) Find an antiderivative $F(x)$ of $f(x) = 8x^3 - 2x^2$ that satisfies $F(-1) = 2$.

(106) A ceramics company determines that the marginal revenue, R', in dollars per unit, from selling the xth vase is given by $R'(x) = x^2 - 1$. Find the total revenue after 3 units were sold.

- *Area and Definite Integrals*

(106) **Theorem.** *Let f be a nonnegative continuous function on $[a, b]$, and let $A(x)$ be the area between the graph of f and the x-axis over $[a, x]$, with $a < x < b$. Then $A(x)$ is a differentiable function of x and $A'(x) = f(x)$.*

(107) Find the area under the graph of $f(x) = 3x^2 + x$ over $[1, 4]$.

(108) **Definition.** Let f be a continuous function on $[a, b]$ and F be any antiderivative of f. Then the **definite integral** of f from a to b is $\int_a^b f(x)\,dx = F(b) - F(a)$.

(109) Evaluate the definite integrals.

(a) $\int_{-2}^{3} (x^2 - 2x + 3)\,dx$

(b) $\int_0^3 e^{-3x}\,dx$

(c) $\int_1^2 \frac{x^4 - x}{x^2}\,dx$

(d) $\int_{-5}^{-1} \frac{1}{x}\,dx$

(110) Northeast Airlines determines that the marginal profit resulting from the sale of x seats on a jet traveling from Atlanta to Kansas City, in hundreds of dollars, is given by $P'(x) = \sqrt{x} - 6$. Find the total profit when 60 seats are sold.

- *Substitution*

(111) Find the integrals.

(a) $\int (x^3 + 1)^4 x^2\,dx$

(b) $\int \frac{1}{5x + 7}\,dx$

(c) $\int x^3 e^{-x^4}\,dx$

(d) $\int_1^e \frac{(\ln x)^2}{x}\,dx$

(e) $\int_0^1 \sqrt{8 - 3x}\,dx$

(f) $\int_0^3 (x - 5)^2\,dx$

- *Consumer Surplus and Producer Surplus*

(112) **Definition.** Let $p = D(x)$ be the demand function for a product. Then the **consumer surplus** for Q units of the product, at a price per unit P, is

$$\int_0^Q D(x)\,dx - QP.$$

Let $p = S(x)$ be the supply function for a product. Then the **producer surplus** for Q units of the product, at a price per unit P, is

$$QP - \int_0^Q S(x)\ dx.$$

The **equilibrium point** is the point at which the supply and demand curves intersect.

(113) In the following problems, $D(x)$ is the price, in dollars per unit, that consumers will pay for x units of an item, and $S(x)$ is the price, in dollars per unit, that producers will accept for x units. Find the equilibrium point, the consumer surplus at the equilibrium point, and the producer surplus at the equilibrium point.
(a) $D(x) = -\frac{5}{6}x + 9$, $S(x) = \frac{1}{2}x + 1$
(b) $D(x) = (x-4)^2$, $S(x) = x^2 + 2x + 6$

§4.5* (optional*) Substitute formula: Let f be continuous in (a, b) and let g be continuous and differentiable in (a, b). Then the substitution formula holds:

$$\int f(g(x))g'(x)dx = \int f(u)du$$

where $u = g(x)$, $du = g'(x)dx$ is the substitution.
For definite integral, we have

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

where $u = g(x)$, $du = g'(x)dx$ is the substitution.