Chapter R. Functions, Graphs, and Models
R.4 Slope and Linear Functions
R.5* Nonlinear Functions and Models
R.6 Exponential and Logarithmic Functions
R.7* Mathematical Modeling and Curve Fitting

- *Linear Functions*

(11) Graph the following equations. Determine if they are functions.
   (a) $y = 2$
   (b) $x = 2$
   (c) $y = 3x$
   (d) $y = -2x + 4$

(12) **Definition.** The variable y is **directly proportional** to x (or **varies directly** with x) if there is some positive constant m such that $y = mx$. We call m the **constant of proportionality**, or **variation constant**.
(13) The weight M of a person's muscles is directly proportional to the person's body weight W. It is known that a person weighing 200 lb has 80 lb of muscle.
   (a) Find an equation of variation expressing M as a function of W.
   (b) What is the muscle weight of a person weighing 120 lb?

(14) **Definition.** A **linear function** is any function that can be written in the form $y = mx + b$ or $f(x) = mx + b$, called the **slope-intercept equation** of a line. The constant m is called the **slope**. The point $(0, b)$ is called the ***y*-intercept**.
(15) Find the slope and y-intercept of the graph of $3x + 5y - 2 = 0$.
(16) Find an equation of the line that has slope 4 and passes through the point $(-1, 1)$.
(17) **Definition.** The equation $y - y_1 = m(x - x_1)$ is called the **point-slope equation** of a line. The point is (x_1, y_1), and the slope is m.

(18) Find the point-slope equation of Problem 16. Compare the two equations.

(19) **Theorem.** The slope of a line passing through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}.$$

Slope can also be considered as an **average rate of change**.

(20) Find the slope of the line passing through the points $(3, -2)$ and $(1, 4)$. Then find the equation of the line.
(21) A skateboard ramp is 2 ft high and 5 ft long in base. Find its slope.
(22) The tuition and fees at public two-year colleges were \$2063 in 2008 and \$3264 in 2014. Find the average rate of change.

(23) A computer firm is planning to sell a new graphing calculator. For the first year, the fixed costs for setting up the new production line are $100,000. The variable costs for each calculator are $20. The sales department projects that 150,000 calculators will be sold during the first year at a price of $45 each.
  (a) Find the total cost $C(x)$ of producing x calculators, the total revenue $R(x)$ from the sale of x calculators, and the total profit $P(x)$ from the production and sale of x calculators.
  (b) How many calculators must the firm sell in order to break even?
  (c) What profit or loss will the firm realize if the expected sale of 150,000 calculators occurs?

- *Quadratic Functions*

(24) A **quadratic function** f is given by $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of a quadratic function is called a **parabola**. The **line of symmetry** of the graph is $x = -\frac{b}{2a}$, and the **vertex** is $\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$.

(25) Find the vertex and line of symmetry of $f(x) = -2x^2 - 4x + 2$. Then graph the function.

(26) **The Quadratic Formula.** *The solutions (also called zeros or roots) of any quadratic equation* $ax^2 + bx + c = 0$, $a \neq 0$, *are given by* $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

(27) Solve the equation $x^2 - 3x + 2 = 0$.

(28) **Definition.** A **polynomial function** f is given by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0,$$

where n is a nonnegative integer (called the **degree**) and $a_n, a_{n-1}, \cdots, a_1, a_0$ are real numbers (called the **coefficients**).

(29) **Definition.** Functions given by the quotient, or ratio, of two polynomials are called **rational functions**.

(30) Graph $f(x) = 1/x$.

(31) **Definition.** y is **inversely proportional** to x (or **varies inversely** with x) if there is some positive number k for which $y = k/x$.

R 6. **Exponential and logarithmic functions.** Let $e = 2.718281828459045$ and $a > 0$.

$$y = e^x \quad \text{if and only if} \quad x = \log_e y$$
$$y = a^x \quad \text{if and only if} \quad x = \log_a y.$$

Ex. Simplify (i) $\log_2(256)$; (ii) $\ln(10e^7)$

Ex. Solve $10^{x+2} = \frac{1}{1{,}000{,}000}$

Ex. The logistic regression function provides an epidemiology model:

$$f(x) = \frac{a}{1 + be^{kx}}$$

where the parameters $a = 243570$, $b = 287.999$, $k = -.31601$. Use this function to give an approximate output value if the input x is equal to 15, 20, 30, 45 and 105.

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