MATH 1441, Review Exam

NAME ____________________ **Id** _________________________

Read each question carefully. **Show all your work** to support your answer. Avoid simple mistakes.

1 Write the equation for the tangent line and the normal line to the graph of

a) $f(x) = x^2 + 4x + 2$ at the Point $P(1, 7)$.

Tangent line:__________________ Normal line:__________________

b) $y = x\cos(x)$ at the point $P(\frac{\pi}{2}, 0)$.

Tangent line:__________________ Normal line:__________________

c) Find the slope of the tangent line to the graph of the equation $xy^3 + x^2y = 10$ at the point $(1, 2)$. The slope is:

2 Find the limits

a) $\lim_{x\to 2} \frac{x^3 - 7x + 6}{x^2 - x - 2} =$ _______________

b) $\lim_{x\to 0} \frac{\sin 3x}{5x} =$ _______________

c) $\lim_{x\to 0} \frac{\tan x}{x}$

d) $\lim_{x\to\infty} \sqrt{x^2 + x + 1} - x$

e) $\lim_{x\to\infty} \frac{7x^2 + 9}{-5x + 2}$

3 Find the maximum and minumum values of the function

a) $f(x) = x^2 - \frac{4}{x^2}$ on the closed interval $[1, 3]$. The maximum is:_______________

The minimum value is: _______________

b*) $f(x) = |x^2 - 1| + \frac{1}{2}x^2$ on the closed interval $[0, 2]$.

The maximum is:__________ The minimum value is: __________

c) $f(x) = -3x^{2/3}$ on $[-1, 1]$

d) Find the intervals on the x-axis on which the funtion $h(x) = \frac{x+1}{x^2+2x+2}$, $(-\infty, \infty)$ is increasing as well as those on which it is decreasing.

4 Find the derivatives of the following functions

a) $f(x) = \dfrac{1}{x} + x\sin(x^2)$. $\quad f'(x) =$

b) $h(x) = (5\sqrt{x} + 1)^{10}$. $\quad h'(x) =$
c) $g(x) = \tan(x + \pi)$

5 Evaluate the following antiderivatives:
a) $\displaystyle\int \left(4 + x^{3/2} - \frac{1}{x^4}\right) dx =$

b) $\int x^2 \cos x^3 dx$
c) $\int \sin^2 x dx$
d) $\int \sec x \ \tan x \ =$ ________ $+ C$
e) $\displaystyle\int \tan x \ \sec^7 x \, dx \ =$

6 Evaluate the following integrals:

a) $\displaystyle\int_0^3 (1 - 3x)^5 \, dx =$

b) $\int_0^{\pi/2} (\sin x)^2 \cos x \, dx \ =$

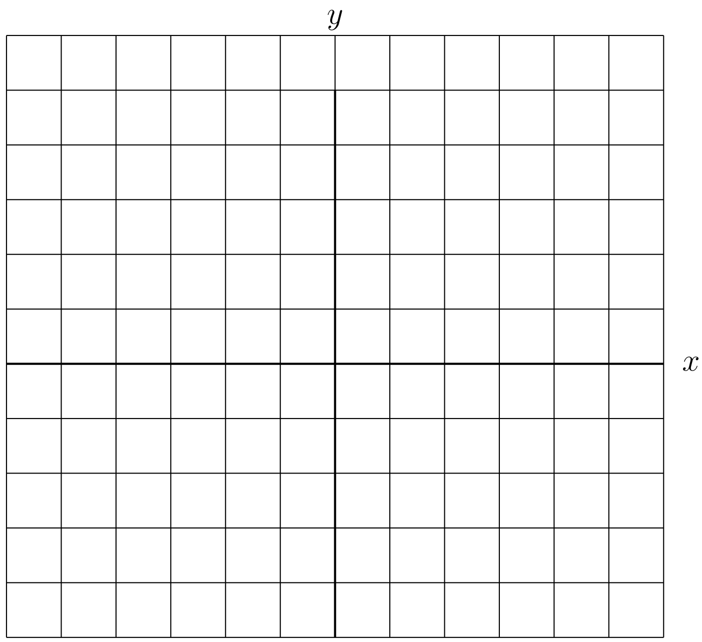
c*) (bonus) $\int_0^1 \frac{1}{\sqrt{x^2+1}} dx$

d) $\int_{-\pi/8}^{\pi/8} \sec(2x) \tan^3(2x) dx$

e) $\int_{-\pi}^{\pi} \sin^2 x \cos^2 x dx$
f) $\displaystyle\int_{x=\frac{-1}{\sqrt{3}}}^{x=0} \frac{dx}{1 + 9x^2} \ =$

7* (bonus) Evaluate the sum $\sum_{j=1}^{20} \left(j^2 - 2j + 1\right) =$ ____________

8 Sketch the graph of the function $f(x) = \dfrac{x^3}{x^2-1}$. Determine the domain and range. If there are any, then identify and label all extrema ([4P]), inflection points ([4P]), intercepts ([4P]), and asymptotes ([4P]). Indicate the concave structure clearly ([4P]).



9* (optional) Use linear approximation to estimate $28^{2/3} \approx$ ________________

10 Sketch the region bounded by the curves and find its area.
a) $y = x^3$ and $y = 2x$ The area is : ________________
b*) *Astroid* given by $x = a\cos^3 t$ and $y = a\sin^3 t$ for $0 \le t \le 2\pi$.

c) Find a vertical line $x = k$ that divides the area enclosed by

$$x = \sqrt{y} \quad \text{and} \quad x = 2 \quad \text{and} \quad y = 0$$

into two equal parts. ANSWER: the vertical line is $x = 60$.

11 a) Sate the Fundamental Theorem of Calculus (two parts). Give an example for each part to illustrate your statement.

b) Evaluate

$$\frac{d}{dx} \int_{17}^{x} \sqrt{t+5}\, dt \;=$$

c)

$$\frac{d}{dx} \int_{1}^{x^3} \cos(y^2)\, dy$$

12. L'Hopital's Rule. Determine whether the limit exists, if so, find the limit.

$$a)\ \lim_{t \to 0} \frac{t - \sin t}{\tan t}$$

$$b)\ \lim_{y \to 2} \frac{y^2 + 6}{y - 2}$$

$$c)\ \lim_{z \to 1} \frac{z^2 + 4z - 5}{z^3 - 1}$$

$$d^*)\ \lim_{r \to 0} (1 + 3r)^{\frac{1}{r}}$$

13. a) Find the volume of the solid that is generated by rotating the plane region bounded by the curves $y = x$ and $y = x^2$ around the y-axes.

The volume is: ________________

b) Give the integral expressions of the volume V of the object (called *Gabriel cone*) generated by revolving the curve from 1 to 100

$$y = 1/x, \qquad 1 \le x < \infty$$

around the x axis.

c) Let R be the region enclosed by

$$y = x^2 \quad \text{and} \quad x = 2 \quad \text{and} \quad y = 0\ .$$

Let V be the volume of the solid obtained by revolving the region R about the line $x = 3$.

d) Make a rough sketch below of the region R, labeling the important points. Using the disk/washer or cylindrical shell method, express the volume V as an integral (or maybe 2 integrals). You do NOT have to evaluate the integral(s).

Let R be the region enclosed by

$$y = 9 - x^2 \quad \text{and} \quad y = 0 \ .$$

Let V be the volume of the solid obtained by revolving the region R about the x-axis.

14 Is the following function continuous at $x = x_0$? If not, determine whether it is a removable or essential discontinuity.
Also, is the function $g(x)$ differentiable at 0?

$$g(x) = \begin{cases} x \cos \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases} \qquad (x_0 = 0)$$

15 a) Explain if the equation $x^3 + x + 1$ has a solution in the interval $[-1, 0]$ or not.

b) Show the following functions have exactly one zero in the given interval.
$f(x) = x^4 + 3x + 1$, $[-2, -1]$ (EX 4.2, Prob. 15)

c) $r(\theta) = \tan\theta - \cot\theta - \theta$, $(0, \pi/2)$ (EX 4.2, Prob. 22)
(Hints for b), c) Rolle's Theorem)

16* (optional) Suppose that the fish population $P(t)$ in a lake is given by the differential equation $\frac{dP}{dt} = -k\sqrt{P(t)}$. If there were 576 fish in the lake at $t = 0$ and 4 weeks later only 144, how many are there after 5 weeks and how long will it take all the fish in the lake to die?

Suggestion: For some of the problems you might want to read the Text or talk to your tutor or Instructor. But most of the problems you should try to work out independently first.