Math 1441 (Calc I) Sections 1.1, 1.2, 1.5, & 1.6 Instructor: Dr. Shijun Zheng
01/14/2020

Course website

§1.1 Functions and Their Graphs

Absolute Value Function. $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Example 4. The function

$$f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

is just one function whose domain is the entire set of real numbers.

§1.2 Combining Functions; Shifting and Scaling Graphs

Definition. If f and g are functions, the **composite** function $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$. The domain of $f \circ g$ consists of the numbers x in the domain of g for which $g(x)$ lies in the domain of f.

Example 2. If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find (a) $(f \circ g)(x)$ (c) $(f \circ f)(x)$.

Exercise 16.* Evaluate each expression using the functions

$$f(x) = 2 - x, \qquad g(x) = \begin{cases} -x, & -2 \leq x < 0 \\ x - 1, & 0 \leq x \leq 2. \end{cases}$$

(a) $f(g(0))$ (b) $g(f(3))$ (c) $g(g(-1))$.

§1.5 Exponential Functions

Definition. If $a \neq 1$ is a positive constant, the function $f(x) = a^x$ is the **exponential function with base *a***.

Rules for Exponents. If $a > 0$ and $b > 0$, the following rules hold for all real numbers x and y.

1. $a^x \cdot a^y = a^{x+y}$ **2.** $\dfrac{a^x}{a^y} = a^{x-y}$ **3.** $(a^x)^y = (a^y)^x = a^{xy}$ **4.** $a^x \cdot b^x = (ab)^x$ **5.** $\dfrac{a^x}{b^x} = \left(\dfrac{a}{b}\right)^x$

Example 2. We use the rules for exponents to simplify some numerical expressions.

1. $3^{1.1} \cdot 3^{0.7}$ **2.** $\frac{(\sqrt{10})^3}{\sqrt{10}}$ **3.** $(5^{\sqrt{2}})^{\sqrt{2}}$ **4.** $7^\pi \cdot 8^\pi$ **5.** $(\frac{4}{9})^{1/2}$

Definition. For every real number x, we define the **natural exponential function** to be $f(x) = e^x$, where $e = 2.71828182845904523536028747135 27...$ is an irrational number.

§1.6 Inverse Functions and Logarithms

Definition. A function $f(x)$ is **one-to-one** on a domain D if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$ in D.

Example 1. (a) $f(x) = \sqrt{x}$ is one-to-one on any domain of nonnegative numbers.
(b) $g(x) = \sin x$ is not one-to-one on the interval $[0, \pi]$. It is one-to-one on $[0, \frac{\pi}{2}]$.

Definition. Suppose that f is a one-to-one function on a domain D with range R. The **inverse function** f^{-1} is defined by $f^{-1}(b) = a$ if $f(a) = b$. The domain of f^{-1} is R and the range of f^{-1} is D.

Example 2. Suppose a one-to-one function $y = f(x)$ is given by a table of values

$\boldsymbol{x}$	1	2	3	4	5	6	7	8
$\boldsymbol{f(x)}$	3	4.5	7	10.5	15	20.5	27	34.5

A table for the values of $x = f^{-1}(y)$ is

$\boldsymbol{y}$	3	4.5	7	10.5	15	20.5	27	34.5
$\boldsymbol{f^{-1}(y)}$	1	2	3	4	5	6	7	8

Example 3. Find the inverse of $y = \frac{1}{2}x + 1$, expressed as a function of x.

Example 4. Find the inverse of the function $y = x^2$, $x \geq 0$, expressed as a function of x.

Definition. The **logarithm function with base *a***, written $y = \log_a x$, is the inverse of the base a exponential function $y = a^x$ $(a > 0, a \neq 1)$. The function $\log_e x$ is called the **natural logarithm function** and is written as $\ln x$.

Theorem 1. For any numbers $b > 0$ and $x > 0$, the natural logarithm satisfies the following rules:

1. Product Rule: $\ln(bx) = \ln b + \ln x$

2. Quotient Rule: $\ln \dfrac{b}{x} = \ln b - \ln x$

3. Reciprocal Rule: $\ln \dfrac{1}{x} = -\ln x$

4. Power Rule: $\ln x^r = r \ln x$

Example 5. We use the properties in Theorem 1 to simplify three expressions.
(a) $\ln 4 + \ln \sin x$ (b) $\ln \dfrac{x+1}{2x-3}$ (c) $\ln \frac{1}{8}$

Property.

(1) The inverse of e^x is given by $y = \ln x$.
(2) The inverse of a^x is given by $y = \log_a x$ for $a > 0$.
(3) Every exponential function is a power of the natural exponential function:
$a^x = e^{x \ln a}$.

Change of Base Formula. Every logarithmic function is a constant multiple of the natural logarithm: $\log_a x = \dfrac{\ln x}{\ln a} \quad (a > 0, a \neq 1)$.

Definition. $\boldsymbol{y = \sin^{-1} x = \arcsin x}$ is the number in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ for which $\sin y = x$.
$\boldsymbol{y = \cos^{-1} x = \arccos x}$ is the number in $[0, \pi]$ for which $\cos y = x$.

Example 8. Evaluate (a) $\arcsin(\frac{\sqrt{3}}{2})$ and (b) $\arccos(-\frac{1}{2})$.