Theorem 2.1. If A, B and C are $m \times n$ matrices, and c and d are scalars, then the properties below are true.
1. $A + B = B + A$
2. $A + (B + C) = (A + B) + C$
3. $(cd)A = c(dA)$
4. $1A = A$
5. $c(A + B) = cA + cB$
6. $(c + d)A = cA + dA$

Theorem 2.2. If A is an $m \times n$ matrix and c is a scalar, then the properties below are true.
1. $A + O_{mn} = A$
2. $A + (-A) = O_{mn}$
3. If $cA = O_{mn}$, then $c = 0$ or $A = O_{mn}$.

Example 2. Solve for X in the equation $3X + A = B$, where $A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}$.

Theorem 2.3. If A, B and C are matrices (with sizes such that the matrix products are defined), and c is a scalar, then the properties below are true.
1. $A(BC) = (AB)C$
2. $A(B + C) = AB + AC$
3. $(A + B)C = AC + BC$
4. $c(AB) = (cA)B = A(cB)$

Example 4. Show that AB and BA are not equal for the matrices $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$.

Example 5. Show that $AC = BC$.

$$A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$$

Definition. The **identity matrix of order n** is the square matrix that has 1's on the main diagonal and 0's elsewhere:

$$I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Theorem 2.4. If A is a matrix of size $m \times n$, then the properties below are true.
1. $AI_n = A$
2. $I_m A = A$

Example 6. (a) Find $\begin{bmatrix} 3 & -2 \\ 4 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Example 7. Find A^3 for the matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$.

Definition. The **transpose** of a matrix is formed by writing its rows as columns.

Example 8. Find the transpose of each matrix.

(b) $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ (d) $D = \begin{bmatrix} 0 & 1 \\ 2 & 4 \\ 1 & -1 \end{bmatrix}$

Theorem 2.6. If A and B are matrices (with sizes such that the matrix operations are defined) and c is a scalar, then the properties below are true.
1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$
3. $(cA)^T = cA^T$
4. $(AB)^T = B^T A^T$