§4.1 Vectors in R^n

Example 1. (a) Represent $\mathbf{u} = (2, 3)$ in the plane.

Example 2. Find the vector sum $\mathbf{u} + \mathbf{v}$ for (a) $\mathbf{u} = (1, 4)$ and $\mathbf{v} = (2, -2)$.

Example 3. Let $\mathbf{v} = (-2, 5)$ and $\mathbf{u} = (3, 4)$. Perform each vector operation.

(a) $\frac{1}{2}\mathbf{v}$ (b) $\mathbf{u} - \mathbf{v}$ (c) $\frac{1}{2}\mathbf{v} + \mathbf{u}$

Definition. Let $\mathbf{u} = (u_1, u_2, u_3, \cdots, u_n)$ and $\mathbf{v} = (v_1, v_2, v_3, \cdots, v_n)$ be vectors in R^n and let c be a real number. The **sum** of $\mathbf{u}$ and $\mathbf{v}$ is the vector

$$\mathbf{u} + \mathbf{v} = (u_1 + v_1, u_2 + v_2, u_3 + v_3, \cdots, u_n + v_n)$$

and the **scalar multiple** of $\mathbf{u}$ by c is the vector

$$c\mathbf{u} = (cu_1, cu_2, cu_3, \cdots, cu_n).$$

Example 4. Let $\mathbf{u} = (-1, 0, 1)$ and $\mathbf{v} = (2, -1, 5)$ in R^3. Perform each vector operation.

(a) $\mathbf{u} + \mathbf{v}$ (b) $2\mathbf{u}$ (c) $\mathbf{v} - 2\mathbf{u}$

Theorem 4.2. Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in R^n, and let c and d be scalars.

1. $\mathbf{u} + \mathbf{v}$ is a vector in R^n.
2. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
3. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
4. $\mathbf{u} + \mathbf{0} = \mathbf{u}$
5. $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
6. $c\mathbf{u}$ is a vector in R^n.
7. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
8. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
9. $c(d\mathbf{u}) = (cd)\mathbf{u}$
10. $1(\mathbf{u}) = \mathbf{u}$

Example 5. Let $\mathbf{u} = (2, -1, 5, 0)$, $\mathbf{v} = (4, 3, 1, -1)$, and $\mathbf{w} = (-6, 2, 0, 3)$ be vectors in R^4. Find $\mathbf{x}$ using equation (b) $3(\mathbf{x} + \mathbf{w}) = 2\mathbf{u} - \mathbf{v} + \mathbf{x}$.

[ans: $\mathbf{x} = \frac{1}{2}(2\mathbf{u} - \mathbf{v} - 3\mathbf{w}) = (9, -\frac{11}{2}, \frac{9}{2}, -4)$]

Theorem 4.3. Let $\mathbf{v}$ be a vector in R^n, and let c be a scalar. Then the properties below are true.

1. The additive identity is unique. That is, if $\mathbf{v} + \mathbf{u} = \mathbf{v}$, then $\mathbf{u} = \mathbf{0}$.
2. The additive inverse of $\mathbf{v}$ is unique. That is, if $\mathbf{v} + \mathbf{u} = \mathbf{0}$, then $\mathbf{u} = -\mathbf{v}$.
3. $0\mathbf{v} = \mathbf{0}$ 4. $c\mathbf{0} = \mathbf{0}$ 5. If $c\mathbf{v} = \mathbf{0}$, then $c = 0$ or $\mathbf{v} = \mathbf{0}$. 6. $-(-\mathbf{v}) = \mathbf{v}$

§4.2 Vector Spaces

Definition. Let V be a set on which two operations (**vector addition** and **scalar multiplication**) are defined. If the listed axioms are satisfied for every $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in V and every scalar c and d, then V is a **vector space**.
1. $\mathbf{u}+\mathbf{v}$ is in V. 2. $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$ 3. $\mathbf{u}+(\mathbf{v}+\mathbf{w})=(\mathbf{u}+\mathbf{v})+\mathbf{w}$
4. V has a **zero vector** $\mathbf{0}$ such that for every $\mathbf{u}$ in V, $\mathbf{u}+\mathbf{0}=\mathbf{u}$.
5. For every $\mathbf{u}$ in V, there is a vector in V denoted by $-\mathbf{u}$ such that $\mathbf{u}+(-\mathbf{u})=\mathbf{0}$.
6. $c\mathbf{u}$ is in V. 7. $c(\mathbf{u}+\mathbf{v})=c\mathbf{u}+c\mathbf{v}$ 8. $(c+d)\mathbf{u}=c\mathbf{u}+d\mathbf{u}$
9. $c(d\mathbf{u})=(cd)\mathbf{u}$ 10. $1(\mathbf{u})=\mathbf{u}$

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