Review Test 1 **Name**
Math 2242 **Id**

Read each question carefully. Avoid making simple mistakes! Put your name and the question number on each page. Put a box around the final answer to a question (Use the back of the page if necessary). You must **show your work** to support your answer in order to get full credits.

1. Show that the given function has an inverse and find the domain and range for the inverse function:
(i) $f(x) = 3x - \cos 2x,\ -\infty < x < \infty$
Evaluate $(f^{-1})'(-1)$.
(ii) $g(x) = (x+2)^3, -\infty < x < \infty$
Find g^{-1} and evaluate $(g^{-1})'(0)$.

2.Evaluate the integrals:
(i) $\int \tan^{-1}(1-x)\,dx$
(ii) $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{3-\sin^2 x}}\,dx$
(iii)* $\int \frac{x^2+4x+4}{x^3+2x}dx$
(iv) $\int \frac{e^x}{\sqrt{1 - e^{(2x)}}}\,dx =$
(v) $\int x^3\ 17^{(x^4)}\,dx =$

3. Find the following limits:

$(i)\quad \lim_{x\to 0} \frac{\sin^{-1} x}{x}$ $\qquad (ii)\quad \lim_{x\to +\infty} x^2 e^{-(x+2)}$

$(iii)\quad \lim_{x\to 0^+} \tan^{-1}(\pi/x)$ $\qquad (iv)\quad \lim_{x\to\infty} \frac{x^5 - 2x^3 + 15x}{x^5 - 7x^3}$

$(v)\quad \lim_{x\to 0^+} (|\ln x|)^x$ $\qquad (vi)\quad \lim_{x\to\infty} (1+\frac{3}{x})^{5x}$

$(vii)\quad \lim_{x\to\infty} \frac{(\ln x)^2}{x+1}$ $\qquad (viii)\quad \lim_{x\to 0} \frac{4x(1-\cos 2x)}{7x - \sin 7x}.$

4. Fill in the blanks (each worth 1 point).
a. $\int \frac{du}{u} =$ _________ $|u| + C$
b. If a is a constant and $a > 0$ but $a \neq 1$, then

$\int a^u\,du =$ ____ $+ C$
c. $\int \sec^2 u\,du =$ ____ $+ C$
d. $\int \sec u\ \tan u\,du =$ _______________________________________ $+ C$
e.

$\int \sin u \, du \; = $ ____ $+C$

f. $\int \cot u \, du \; = $ ____ $+C$

g. $\int \sec u \, du \; = $ ____ $+C$

h. If a is a contant and $a > 0$ then

$\int \frac{1}{\sqrt{a^2-u^2}} \, du \; = $ ____ $+C$

i. If a is a contant and $a > 0$ then

$\int \frac{1}{a^2+u^2} \, du \; = $ ____ $+C$

j. The integral of $y = f(x)$ <u>with respect to x</u> is denoted by $\int f(x)\, dx$.

The integral of $x = g(y)$ <u>with respect to y</u> is denoted by ____________________ .

k. $\cos^{-1}(\frac{\sqrt{2}}{2}) = $ __________.

l. $\csc^{-1}(-\frac{2}{\sqrt{3}}) = $ __________.

m. $\tan^{-1}(-1) = $ __________.

5. Let R be the region enclosed by

$$y = x^2 \qquad \text{and} \qquad y = x \; + \; 2 \; .$$

Let A be the area of the region R.

a. The points of intersection of $y = x^2$ and $y = x + 2$ are $P =$ (_____ , _____) and $Q =$ (_____ , _____).

Make a rough sketch of the region R, labeling P and Q.

b. Express the area A as integral(s) with respect to x (so you want dx).
You do NOT have to evaluate the integral(s) nor do lots of algebra.

A =

c. Express the area A as integral(s) with respect to y (so you want dy).
You do NOT have to evaluate the integral(s) nor do lots of algebra.

A =

6. Let $a, b > 0$ be constants. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

The area = ____________________.

7. Evaluate the integrals.

a) $\displaystyle\int \frac{1}{x(\ln x)^{3/2}}\,dx$ b) $\displaystyle\int_0^1 \frac{1}{\sqrt{x^2+1}}dx$

c) $\displaystyle\int xe^{-2x}dx$ d) $\displaystyle\int_0^{\pi/4} e^x \sin(x)dx$

e*) $\displaystyle\int xe^x \cos(x)dx$ f*) $\displaystyle\int \sec^3 x dx$

g) $\displaystyle\int_{-\pi/12}^{\pi/12} \tan(3x)dx$ h) $\displaystyle\int \sin^2 x dx$

i) $\displaystyle\int \sin(\ln\theta)d\theta$ j) $\displaystyle\int \cot^{-1}(8y)dy$

k)* $\displaystyle\int_0^8 \frac{x^3}{x^2+16x+64}dx$ l) $\displaystyle\int \frac{18dx}{(81x^2+1)^2}$.

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Answers to Review Test 1

1. (i) $f'(x) = 3+2\sin(2x) \geq 1 > 0$ means that f is increasing and so the inverse function f^{-1} exists. The domain of f^{-1} is equal to the range of f; the range of f^{-1} equal to the domain of f. Hence domain of f^{-1} is $(-\infty,\infty)$, range of f^{-1} is $(-\infty,\infty)$.

3 (vi) From the identity $\lim_{x\to\infty}(1+\frac{1}{x})^x = e$ we know that $\lim_{x\to\infty}(1+\frac{3}{x})^x = e^3$. Hence

$$\lim_{x\to\infty}(1+\frac{3}{x})^{5x} = \lim_{x\to\infty}\left((1+\frac{3}{x})^x\right)^5 = (e^3)^5 = e^{15}.$$

5 (a) Solve the equations $\begin{cases} y = x^2 \\ y = x+2 \end{cases}$ to obtain $P=(-1,1), Q=(2,4)$

(b) $A = \int_{-1}^2 (x+2-x^2)dx$.

7. (b) Sub $x = \tan t$, then $dx = \sec^2 t dt$. We have $x^2+1 = \sec^2 t$.

$$\int_0^1 \frac{1}{\sqrt{x^2+1}}dx = \int_0^{\pi/4} \frac{\sec^2 t}{\sec t}dt$$
$$= \int_0^{\pi/4} \sec t dt = \ln|\sec t + \tan t||_0^{\pi/4} = \ln(\sqrt{2}+1).$$

(d) $\displaystyle\int_0^{\pi/4} e^x \sin(x)dx = \frac{1}{2}e^x(\sin x - \cos x)|_0^{\pi/4} = \frac{1}{2}$.

(j) Let $u = \cot^{-1}(8y), dv = dy$, then $du = -8dy/(1+64y^2), v = y$. We have

$$\int \cot^{-1}(8y)dy = y\cot^{-1}(8y) + \int \frac{8y}{1+64y^2}dy$$

For the integral on the right hand side, use substitution $w = 1+64y^2$, then $dw = 128ydy$, so we get

$$\int \frac{8ydy}{1+64y^2} = \int \frac{8\,dw/128}{w} = \frac{1}{16}\ln|w| + C$$

Hence we obtain

$$\int \cot^{-1}(8y)dy = y\cot^{-1}(8y) + \frac{1}{16}\ln|1+64y^2| + C.$$

7. (e*). Let $I = \int xe^x\cos(x)dx$. Integrating by parts, we get

$$\begin{aligned} I &= xe^x\cos x - \int e^x\cos x dx + \int xe^x \sin x dx \\ &= xe^x\cos x - \int e^x\cos x dx + xe^x\sin x - \int e^x\sin x dx - \int xe^x\cos x dx \\ &= xe^x\cos x - \int e^x\cos x dx + xe^x\sin x - \int e^x \sin x dx - I \end{aligned}$$

Hence

$$2I = xe^x\cos x + xe^x\sin x - \int e^x\cos x dx - \int e^x \sin x dx$$

From the older exercises or the table of the text, we know that

$$\int e^x\cos x dx = \frac{1}{2}e^x(\sin x + \cos x)$$
$$\int e^x\sin x dx = \frac{1}{2}e^x(\sin x - \cos x)$$

Finally we obtain

$$\begin{aligned} 2I &= xe^x\cos x + xe^x\sin x - \frac{1}{2}e^x(\sin x + \cos x) - \frac{1}{2}e^x(\sin x - \cos x) + C \\ &= xe^x\cos x + xe^x\sin x - e^x\sin x + C \end{aligned}$$

You can also use the interactive integration tool in our course website to check that this answer is absolutely correct!