Review Exam 3 **Name**
Math 2243 **Id**

Read each question carefully. Avoid simple mistakes. Put a box around the final answer to a question (use the back of the page if necessary). For full credit you must *show your work*. You must have enough written work, including explanations when called for, to justify your answers. Incomplete solutions may receive partial credit if you have written down a reasonable partial solution.

(1) (a) Compute $\int_0^9 \int_{2\sqrt{x}}^6 xy\,dydx$.
(b) Sketch the region in the xy-plane which is described by the integral in part (a). Rewrite the integral as an iterated integral using the order $dxdy$ (*DO NOT evaluate the integral*)
(c) Give a geometric or physical interpretation of the meaning of the integral in part (a).

(2) Evaluate by sketching the planar region and converting to polar coordinates: $\int_0^3 \int_0^{\sqrt{9-x^2}} (x^2+y^2)^{3/2} dydx$.

(3) Find the volume of the tetrahedron enclosed by the planes: $x+2y+z=2$, $x=0$, $z=0$, and $y=2x$.

(4) * Evaluate the line integral: $\int_C xy^3 ds$, C is the left half of the circle $x^2+y^2=4$.

(5) Fill in the blanks:
(a) $\iint_R 1dA =$
(b) $\iiint_D 1dV =$
(c) Give a physical interpretation of the integral $\iiint_D f(x,y,z)dV$.
(d) Without doing any integration, tell the volume in the first octant under the surface $x+6y+2z=60$.

(6) Compute $\int_0^3 \int_0^2 \int_0^{10x} (2y+z)dzdydz$.

(7) Set up an iterated triple integral to compute the volume under the surface $2x+6y+3z=18$ in the first octant. Make and use a good sketch. (Do not evaluate the integral)

(8) Evaluate the triple integral.
(a) $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} z^3 dzdydx$.
(b) $\int_0^1 \int_0^z \int_0^y ze^{-y^2} dxdydz$.

(9) Identify, sketch and describe the graph of each equation in three dimensions:
(a) $\rho = 4$
(b) $r = 2$
(c) $\phi = \pi/3$

(10) Using spherical coordinates to find the volume of the ice cream cone cut from the solid sphere $\rho \leq 5$ by the cone $\phi = \pi/4$.

(11) Set up a triple integral by identifying the region and limits but *do not evaluate it.*
(a) $\iiint_R (x^3+xy)dV$ in cylindrical coordinates where R is the solid in the first octant that lies beneath the paraboloid $z = 1-x^2-y^2$.

(b) (optional*) Use spherical coordinates for the mass of the ice-cream cone inside the cone $z = \sqrt{x^2 + y^2}$ and topped off by $x^2 + y^2 + z^2 = 1$, with density $\delta(x, y, z) = 4z$.

(12) (a) Evaluate $\int_C (x - y + z + 3)ds$ where C is the line segment given by $\mathbf{r}(t) = t\mathbf{i} + (1 - t)\mathbf{j} + \mathbf{k}$ from $(0, 1, 1)$ to $(2, -1, 1)$.

(b) Give a physical interpretation of the meaning of the result in part (a).

(13) * Determine whether or not $\mathbf{F}(x, y) = (2x \cos y - y \cos x)\mathbf{i} + (-x^2 \sin y - \sin x)\mathbf{j}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

(14) * Use the Fundamental Theorem for line integrals to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along a very complicated curve C from $(1, -1, 0)$ to $(2, 2, -3)$, and $\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + (xy + 2z)\mathbf{k}$.

(15) (optional*)

(a) Evaluate the integral $\iint_R x^2 dA$ where R is the region bounded by $y = -2x$ and $y = 2x - x^2$.

(b) Find the area of one loop of the polar curve $r = \sin 3\theta$ (Hint: You can think of the area formula by Green's Theorem $Area = \frac{1}{2}\oint_C xdy - ydx = \frac{1}{2}\oint_C xdx + ydy$, C a closed curve with counterclockwise orientation).

Solutions:

4 The parametric equation for the left semi-circle is $r(t) = (2\cos t, 2\sin t)$, $t \in [\frac{\pi}{2}, \frac{3\pi}{2}]$. So $s'(t) = |v(t)| = |dr/dt| = |(-2\sin t, 2\cos t)| = 2$. We have

$$\int_C xy^3 ds = \int_{\pi/2}^{3\pi/2} \cos t(\sin t)^3 |v(t)| dt$$
$$= 2\int_{\pi/2}^{3\pi/2} \cos t(\sin t)^3 dt = 0.$$

13 Check that $M_y = N_x \Rightarrow F$ is conservative.

To find f, notice that $\frac{\partial f}{\partial x} = M$,

$$f(x, y) = \int M dx = \int (2x \cos y - y \cos x)dx$$
$$= x^2 \cos y - y \sin x + h(y)$$

To find h(y), taking derivative in y we get

$$\partial_y f(x, y) = N(x, y) \Rightarrow$$
$$= -x^2 \sin y - \sin x + h'(y) = -x^2 \sin y - \sin x$$

$\therefore h'(y) = 0$ and so $h(y) = C$

14 The function F has a potential function (anti-derivative) $f = xyz + z^2$, because $\nabla f = F$. Fundamental Theorem of Calculus tells that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f(2, 2, -3) - f(1, -1, 0) = -3$$