Review Exam **Name**
Math 2243 **Id**

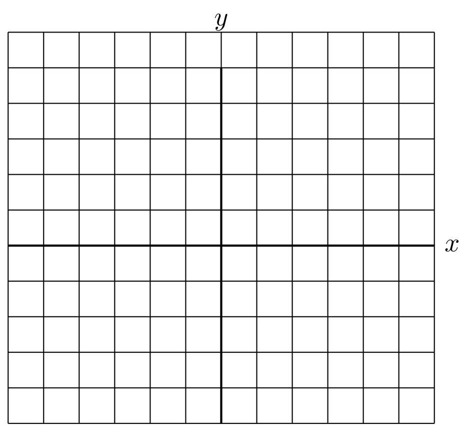
Read each question carefully. Avoid simple mistakes. Put a box around the final answer to a question (use the back of the page if necessary). For full credit you must *show your work*. Incomplete solutions may receive partial credit if you have written down a reasonable partial solution.

(1) Let $u = (1, -3, 2)$ and $v = (-4i - k)$. Find:
  (a) the vector projection of u onto v.
  (b) the angle between u and v.
  (c) the area of the triangle spanned by u and v.
  (d) the equation of the plane through $P(1, 1, 1)$ with normal perpendicular to both u and v.

(2) Let $r(t) = (2\sin t, 5t, 2\cos t)$. Find
  (a) the length of the curve (helix) over $-10 \le t \le 10$.
  (b) the equation of the tangent line to the curve when $t = \pi/4$
  (c) the curvature at any given time t (hint: $\kappa(t) = |r'(t) \times r''(t)|/|r'(t)|^3$)

(3) Find a vector orthogonal to the plane containing the points: $P(1, -1, 0)$, $Q(-2, 3, -4)$, and $R(1, 1, 1)$. Find the equation of the plane. Find the area of the triangle PQR.

(4) Let $h(x, y) = \sqrt{9 - x^2 - y^2}$
  (a) Find and sketch the domain of the function h



  (b) Find the range of $h(x, y)$

(5) Use the definition of continuity to explain whether or not the function $f(x, y)$ is continuous at $(0, 0)$

$$f(x,y) = \begin{cases} \frac{3x^2y}{x^2+2y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(6) (a) Find all critical points, extrema and saddle points of $g(x, y) = x^4 + y^4 - 4xy + 1$.
  (b) Find the absolute maximum and absolute minimum of values of $f(x, y) = x^2 + y^2 + x^2y + 4$ on the region $D = \{(x, y) \mid |x| \le 1,\ |y| \le 1\}$

(7) (a) Use the limit definition to find the partial derivative $\frac{\partial f(x,y)}{\partial y}$ where $f(x,y) = x^2y$.
(b) Find the indicated partial derivatives: f_{xy}, f_{yx}, where $f(x,y) = ye^{\frac{x}{y}}$
(c) Find the directional derivative of $f(x,y,z) = x^2 - x^2y^3 + 10z + 30$ at $(1,1,1)$ in the direction $\mathbf{v} = (2,0,1)$.

(8) Let $w = \cos(x+3y)$, $x = r^2 + s^2$ and $y = rs$
(a) Use the Chain rule to express w_r and w_s as functions of r and s.
(b) Evaluate w_r and w_s at $(r,s) = (0,\pi)$.

(9) Let $g(x,y) = \ln(x^2+y^2)$.
(a) Find the partial derivatives g_x and g_y.
(b) Find the equation of the tangential plane to the surface $z = g(x,y)$ at $(0,1,0)$.
(c) Find the linearization $L(x,y)$ of the function $g(x,y)$ at the point $(0,1)$.
(d) Give an estimate for the value $g(-0.1, 0.9)$.
(e) Calculate $\Delta g := (\partial_x^2 + \partial_y^2)g$.

(10) (a) Find the differential of $u = \sqrt{x^2+y^2+z^2}$.
(b) Find the tangent plane to the surface $z = y\ln x$ at $(1,4,0)$.
(c) Use the Chain rule (plot a tree diagram) to find the indicated partial derivatives $u = x^2 + \sqrt{y^2+z^2}$, $x = \sin(r)\cos(s)$, $y = \sin(r)\sin(s)$, $z = 3$. Find $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial s}$. Your final answer should be given in terms of r and s only.

(11) (a) *(optional) Find the area of the surface for the part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2+y^2 = 1$ and $x^2+y^2 = 4$.
(b) Find the volume of the tetrahedron enclosed by the planes: $x+2y+z = 2$, $x = 0$, $z = 0$, and $y = 2x$.

(12) Quick answer:
(a) The curvature of $x^2+y^2 = 49$ is ____________________
(b) How is the gradient at a point related to the level curve through that point? ____________________
(c) If z depends upon u and v, and u and v depend upon t, then ____________________
(d) The graph of $r(t)$ is a ____________________
(e) The graph of $z = f(x,y)$ is a ____________________ .
(f) The graph of $r(t) = \langle t+1, 2t-1, 3t\rangle$ is ____________________
(g) Given a vector valued function $r(t)$, where does the acceleration lie ?
____

(13) (a) Evaluate the line integral $\int_C xy\,dx + (x+y)\,dy$ along:
(i) That part of the graph of $y = x^2$ from $(-1,1)$ to $(2,4)$
(ii) The line segment from $(-1,1)$ to $(2,4)$
(iii) The broken line from $(-1,1)$ to $(2,1)$ to $(2,4)$.
(b) Evaluate the line integral: $\int_c xy^3\,ds$, C is the left half of the circle $x^2+y^2 = 4$.

(14) (a) Determine whether or not $\mathbf{F}(x,y) = (2x\cos y - y\cos x)\mathbf{i} + (-x^2\sin y - \sin x)\mathbf{j}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

(b) Show that $\mathbf{F}(x,y) = (e^x(\sin(x+3y)+\cos(x+3y)), 3e^x\cos(x+3y))$ is conservative and find a potential function. (hint: verify that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$)

(c) Show that $\mathbf{F}(x,y) = (M,N,P) = (2xyz+3y^2, x^2z+6xy-2z^3, x^2y-6yz^2)$ is conservative and find a potential function. (hint: show that $\overrightarrow{\text{curl}}\,\mathbf{F} = (P_y - N_z, M_z - P_x, N_x - M_y) = \mathbf{0}$

(15) Use the Fundamental Theorem for line integrals to evaluate $\int_c \mathbf{F}\cdot d\mathbf{r}$ along a very complicated curve C from $(1,-1,0)$ to $(2,2,-3)$, and $\mathbf{F}(x,y,z) = yz\mathbf{i} + xz\mathbf{j} + (xy+2z)\mathbf{k}$.

(16) Let T be the unit tangent and N the (outward) unit normal for a curve C: $t \mapsto \mathbf{r}(t)$ on [a,b]. Let R be a simply connected bounded open domain in $\mathbf{R}^2$, where $F = Mi + Nj$ is continuously differentiable.

(a) Scalar line integral on $\mathbf{R}^2$ is:

$$\int_C f(x,y)ds = \int_a^b f(x(t),y(t))|r'(t)|dt \;\; = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

(b) Vector line integral

$$W = \int_C F\cdot T ds = \int_a^b F(r(t))\cdot r'(t)dt \;\; = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

(c) (optional)* Green's Theorem (circulation): $\oint_{\partial R} F\cdot T ds = \oint_{\partial R} M dx + N dy = \iint_R (\nabla\times F)dxdy \;\; = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

(d) (optional)* Green's Theorem (flux): $\oint_{\partial R} F\cdot N ds = \oint_{\partial R} M dy - N dx = \iint_R (\nabla\cdot F)dxdy \;\; = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$.

(e) (optional)* Use Green's Theorem to evaluate the line integral along the given positively oriented curve: $\int_C (y^2+e^{\sqrt{x}})dx + (2x+\cos(y^2))dy$ where C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

(f) (optional)* Use Green's Theorem to evaluate $\int_c \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = < \sqrt[2008]{x} + y^2, x^2 + \frac{1}{1+y^{2008}} >$, C consists of the arc of the curve $y = \sin x$ from $(0,0)$ to $(\pi,0)$ and the line segment from $(\pi,0)$ to $(0,0)$.

(17) (optional*) Let $\mathbf{F}(x,y,z) = e^x \sin y\ \mathbf{i} + e^x\cos y\ \mathbf{j} + z\ \mathbf{k}$.

(a) find curl $\mathbf{F}$ and div $\mathbf{F}$

(b) find div curl $\mathbf{F}$, i.e. the divergence of the vector field curl $\mathbf{F}$

(c) What is the physical meaning of curl $\mathbf{F}$?

(18) (optional)* Find the area of the surface for the part of the sphere $x^2 + y^2 + z^2 = 9$ that lies within the cylinder $(x-2)^2 + y^2 = 1$ and above the xy-plane.

(19) (optional*)

(a) Evaluate the integral $\iint_R x^2 dA$ where R is the region bounded by $y = -2x$ and $y = 2x - x^2$.

(b) Find the area of one loop of the polar curve $r = \sin 3\theta$

(Hint: You can think of the area formula by Green's Theorem $Area = \frac{1}{2}\oint_C xdy - ydx = \frac{1}{2}\oint_C xdx + ydy$, C a closed curve with counterclockwise orientation).

(20) Evaluate the triple integrals:

(a) $\int_{-1}^{1}\int_0^{\sqrt{1-x^2}}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} z^3 dzdydx$.

(b) $\int_0^1\int_0^z\int_0^y ze^{-y^2}dxdydz$.

(21) Set up a triple integral but *do not evaluate it*.

(a) $\iiint_R (x^3+xy)dV$ in cylindrical coordinates where R is the solid in the first octant that lies beneath the paraboloid $z = 1 - x^2 - y^2$.

(b) (optional*) using spherical coordinates for the mass of the ice-cream cone inside the cone $z = \sqrt{x^2+y^2}$ and topped off by $x^2+y^2+z^2 = 1$, with density $\delta(x,y,z) = 4z$.

Solutions:

1. (a) $\frac{\langle 24,0,6\rangle}{\sqrt{17\cdot 17}}$

   (b) $\theta = \cos^{-1}(-\frac{6}{\sqrt{14\cdot 17}}) = 1.97025\,rad = 113°$

   (c) $Area = \frac{1}{2}|u\times v| = \frac{\sqrt{202}}{2}$

   (d) $3x - 7y - 12z = -16$

2. (a) $20\sqrt{29}$

   (b) $r'(t) = (2\cos t, 5, -2\sin t)$, $T(t) = (2\cos t, 5, -2\sin t)/\sqrt{29}$, $\therefore\ \ell(t) = (\sqrt{2}, 5\pi/4, \sqrt{2}) + t(\sqrt{2}, 5, -\sqrt{2})$

   (c) 2/29. Indeed,

$$\kappa = |\frac{dT}{ds}| = \frac{1}{|v|}|\frac{dT}{dt}|$$
$$= \frac{1}{\sqrt{29}}|(-2\sin t, 0, -2\cos t)|/\sqrt{29} = 2/29$$

   where $v = |r'(t)| = \sqrt{4\cos^2 t + 5^2 + 4\sin^2 t} = \sqrt{29}$.

3. Using cross product we find $N = PQ \times PR$ is orthogonal to the plane.

   The equation of the plane is given by the standard form:

$$N\cdot\langle x-x_0, y-y_0, z-z_0\rangle = 0$$

where (x_0,y_0,z_0) can be taken to be any of the three point P, Q, or R.

Now, $PQ = Q - P = (-2,3,-4) - (1,-1,0) = \langle -3,4,-4\rangle$, $PR = R - P = (1,1,1) - (1,-1,0) = \langle 0,2,1\rangle$

$$N = \begin{vmatrix} i & j & k \\ -3 & 4 & -4 \\ 0 & 2 & 1 \end{vmatrix} = 12i + 3j - 6k$$

Hence the equation of the plane is

$$12(x-1) + 3(y-1) - 6(z-1) = 0$$
$$or\ 4x + y - 2z = 3$$

Second method. Let the plane equation be $A(x-1)+B(y-1)+C(z-1) = 0$. Substituting the coordinates of P and Q into this equation to obtain a relation between A, B, C which will tell the three components of N.

4. It is easy to check $2|xy| \le x^2 + y^2 \le x^2 + 2y^2$ Hence

$$|\frac{3x^2y}{x^2+2y^2}| \le \frac{3}{2}|x| \to 0$$

as $(x, y) \to 0$, which agrees with $f(0, 0)$, and we know that $f(x, y)$ is continuous at $(0, 0)$.

5 (a) Saddle at $(0, 0)$, local at $(1, 1)$, $(-1, -1)$.

Indeed, Solve

$$\begin{cases} g_x = 4x^3 - 4y = 0 \\ g_y = 4y^3 - 4x = 0 \end{cases}$$

$$\begin{cases} x^3 - y = 0 \\ y^3 - x = 0 \end{cases}$$

Sub $y = x^3$ into $x = y^3$ to get $x = x^9 \Rightarrow$

$$x = 0, x = \pm 1.$$

6(a) By definition

$$\frac{\partial f(x,y)}{\partial y} = \lim_{k\to 0} \frac{f(x, y+k) - f(x,y)}{k}$$

$$= \lim_{k\to 0} \frac{x^2(y+k) - x^2 y}{k} = \lim_{k\to 0} \frac{x^2(k)}{k}$$

$$= \lim_{k\to 0} x^2 = x^2.$$

6 (b)

$$f_{xy} = f_{yx} = -\frac{x}{y^2} e^{x/y}$$

provided $y \neq 0$.

[6 (c)] $D_u f = \nabla f|_{(1,1,1)} \cdot \frac{u}{|u|} = (0, -3, 10) \cdot \frac{(2,0,1)}{\sqrt{5}} = 2\sqrt{5}$

7 (a) By definition

$$du = u_x dx + u_y dy + u_z dz$$

$$= \frac{x}{\sqrt{x^2+y^2+z^2}} dx + \frac{y}{\sqrt{x^2+y^2+z^2}} dy + \frac{z}{\sqrt{x^2+y^2+z^2}} dz$$

(b) If $z = f(x, y)$, then the graph of this function is a surface. At any point (x_0, y_0, z_0) on the surface, there is a tangent plane whose normal vector is given by $N = \langle f_x, f_y, -1 \rangle$. Then the equation is given by

$$N \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0.$$

(c) The Chain rule reads:

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial r}$$

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial s}$$

8 (a) Let $z = f(x, y) = y^2 - x^2$ and R the region on the xy-plane $\{(x, y) : 1 \leq \sqrt{x^2 + y^2} \leq 2\}$. Then the Area of the surface

$$\begin{aligned} S &= \iint_R \sqrt{1 + f_x^2 + f_y^2}dxdy \\ &= \int_{\theta=0}^{\theta=2\pi} \int_{r=1}^{r=2} \sqrt{1 + 4r^2}rdrd\theta \end{aligned}$$

where $f_x = -2x$, $f_y = 2y$.

(b) The tetrahedron is bounded on the bottom by $y = 2x$, $x = 0$ and $x + 2y = 2$. The volume V can be evaluated over this triangle $\triangle OBC$ by a double integral. Here $O = (0, 0), B = (2/5, 4/5), C = (0, 1)$. Let $z = 2 - x - 2y$, which is the equation of the plane on the top.

$$V = \iint_\triangle zdxdy = \int_{x=0}^{x=2/5} \int_{y=2x}^{y=1-\frac{x}{2}} (2 - x - 2y)dydx.$$

9. (a) 1/7

(b) "perpendicular to the tangent vector at the point, that is $\nabla F \cdot \mathbf{T} = 0$, given that $F(x, y) = C$ is a level curve"
or alternatively, "orthogonal to the level curve".

(c) z is a function of t

(d) curve

(e) surface

(f) line

(g) $\mathbf{a}$ lies in the plane spanned by the tangential and principle normal of $\mathbf{r} = \mathbf{r}(t)$

10 (a) (i) $C(t) = (t, t^2)$, $C'(t) = (1, 2t)$,

$$\begin{aligned} &\int_{-1}^{2} (t^3 + 2t(t + t^2))dt = \int_{-1}^{2} (2t^2 + 3t^3)dt \\ =&(\frac{2}{3}t^3 + \frac{3}{4}t^4)|_{-1}^{2} = 69/4 \end{aligned}$$

(ii) $C(t) = (-1, 1) + (3, 3)t = (-1 + 3t, 1 + 3t)$, $C'(t) = (3, 3)$,

$$\begin{aligned} w &= \int_{-1}^{2} 3(-1 + 3t)(1 + 3t) + (3t + 3t)dt \\ =&3 \int_{-1}^{2} (9t^2 - 1 + 6t)dt = 99 \end{aligned}$$

(iii)

$$\begin{aligned} w &= \int_{-1}^{2} xdx + \int_{1}^{4} (2 + y)dy \\ =&\frac{x^2}{2}|_{-1}^{2} + (2y + \frac{y^2}{2})|_1^4 = 15. \end{aligned}$$

b) The parametric equation for the left semi-circle is $r(t) = (2\cos t, 2\sin t)$, $t \in [\frac{\pi}{2}, \frac{3\pi}{2}]$. So $s'(t) = |v(t)| = |dr/dt| = |(-2\sin t, 2\cos t)| = 2$. We have

$$\int_C xy^3 ds = \int_{\pi/2}^{3\pi/2} \cos t(\sin t)^3 |v(t)| dt$$
$$= 2\int_{\pi/2}^{3\pi/2} \cos t(\sin t)^3 dt = 0.$$

11 a) check that $M_y = N_x \Rightarrow F$ is conservative.
To find f, notice that $\frac{\partial f}{\partial x} = M$,

$$f(x,y) = \int M dx = \int (2x\cos y - y\cos x) dx$$
$$= x^2\cos y - y\sin x + h(y)$$

To find h(y), taking derivative in y we get

$$\partial_y f(x,y) = N(x,y) \Rightarrow$$
$$= -x^2\sin y - \sin x + h'(y) = -x^2\sin y - \sin x$$

$\therefore h'(y) = 0$ and so $h(y) = C$
b) $f(x,y) = e^x \sin(x+3y)$
c) It is conservative because:

$$M_y = 2xz + 6y = N_x = 2xz + 6y$$
$$N_z = P_y = x^2 - 6z^2$$
$$P_x = M_z = 2xy.$$

In order to find the potential function f such that $\nabla f = F$, that is, $f_x = M$, $f_y = N$, $f_z = P$, we integrate

$$f = \int M dx = \int (2xyz + 3y^2) dx = x^2(yz) + (3y^2)x + g(y,z).$$

Then, to recover $g(y,z)$, taking derivative in y both sides we get

$$f_y = N$$
$$or\ x^2 z + (6y)x + g'_y(y,z) = x^2 z + 6xy - 2z^3$$
$$simplify\ g'_y(y,z) = -2z^3$$
$$\Rightarrow g = \int (-2z^3) dy = -2z^3 y + h(z).$$

It remains to recover $h(z)$. To do that we take derivative in z both sides of $f = x^2(yz) + (3y^2)x - 2z^3 y + h(z)$ to obtain

$$f_z = P$$
$$or\ x^2 y - 6z^2 y + h'(z) = x^2 y - 6yz^2$$
$$simplify\ h'(z) = 0 \Rightarrow h(z) = constant.$$

Hence, $f(x,y,z) = x^2 zy + 3xy^2 - 2z^3 y + C$.

12. The function F has a potential function (anti-derivative) $f = xyz + z^2$, because $\nabla f = F$. F.T.C tells that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f(2, 2, -3) - f(1, -1, 0) = -3$$

13. a) $\int_a^b f(r(t))|v(t)|dt$

b) $\int_a^b F(r(t)) \cdot d\mathbf{r} = \int_C Mdx + Ndy$

c) Let $F = \langle M, N \rangle$, then $\nabla \times F = (\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y})\mathbf{k}$.

$$\therefore \quad \iint_R (\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y})dxdy$$

d) $\nabla \cdot F = \langle \partial_x, \partial_y \rangle \cdot \langle M, N \rangle = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$

$$\therefore \quad \iint_R (\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y})dxdy$$

14. (a) 64/5

(b) $\pi/12$

15*. Find the area of the surface for the part of the sphere $x^2 + y^2 + z^2 = 9$ that lies within the cylinder $(x - 2)^2 + y^2 = 1$ and above the xy-plane.

[Solution] The surface $z = f(x, y) = \sqrt{9 - x^2 - y^2}$ is defined over R: $(x - 2)^2 + y^2 = 1$.

$$f_x = \frac{-x}{\sqrt{9 - x^2 - y^2}}, \quad f_y = \frac{-y}{\sqrt{9 - x^2 - y^2}}$$

$$1 + f_x^2 + f_y^2 = \frac{9}{9 - x^2 - y^2}$$

Use polar coordinates $x = r\cos\theta, y = r\sin\theta$ to get

$$\text{Surface Area} = \iint_R \sqrt{1 + f_x^2 + f_y^2}dA$$

$$= \int_{-\pi/6}^{\pi/6} \int_{r=h(\theta)}^{r=g(\theta)} \frac{3}{\sqrt{9 - r^2}} rdrd\theta$$

where $g(\theta) = 2\cos\theta - \sqrt{4\cos^2\theta - 3}$, $h(\theta) = 2\cos\theta + \sqrt{4\cos^2\theta - 3}$. Note that $4\cos^2\theta - 3 \geq 0$ if $-\pi/6 \leq \theta \leq \pi/6$.

19 (a) Area= $\frac{1}{2}\oint_C xdy - ydx$

(b) The curve has three leaves or loops. Because of symmetry, we look at the one in the first quadrant, bounded by $\theta = 0$ and $\theta = \pi/3$. Hence

$$\text{Area of one loop} = \iint dA = \int_0^{\pi/3} \int_0^{\sin 3\theta} rdrd\theta$$

$$= \frac{1}{2}\int_0^{\pi/3} (\sin 3\theta)^2 d\theta = \frac{1}{4}\int_0^{\pi/3} (1 - \cos 6\theta)d\theta = \frac{\pi}{12}$$

17. (a) Hint: sketch a picture. Use cylindrical coordinates to write the integral as

$$\int_0^{\pi} \int_0^1 \int_{r^2}^{r} z^3 dzrdrd\theta = \int_0^{\pi} \int_0^1 \frac{z^4}{4}|_{r^2}^{r} rdrd\theta$$

$$= \frac{1}{4}\int_0^{\pi} \int_0^1 (r^4 - r^8)rdrd\theta = \pi/60.$$

(b) $1/(4e)$. Indeed,

$$\int_0^1 \int_0^z \int_0^y ze^{-y^2} dxdydz = \int_0^1 \int_0^z yze^{-y^2} dydz$$
$$= \quad \text{(use sub } u = y^2 \text{ to finish it)}$$

18. (a) $\int_0^{2\pi} \int_0^1 \int_0^{1-r^2} (r^3 \cos^3\theta + r^2 \cos\theta \sin\theta) dzrdrd\theta$

(b) Spherical coordinates $(x, y, z) = (\rho, \phi, \theta)$

$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases}$$

where $\rho \geq 0, \phi \in [0, \pi], \theta \in [0, 2\pi)$. The side of the cone $z = \sqrt{x^2 + y^2}$ and the z-axis make an angle $\phi = \pi/4$, which is the upper limit for ϕ. The sphere has an equation $\rho = \sqrt{x^2 + y^2 + z^2} = 1$.

Hence the mass of the ice-cream cone

$$M = \iiint \delta dV = \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi/4} \int_0^1 4z\rho^2 \sin\phi d\rho d\phi d\theta$$
$$= \int_0^{2\pi} \int_0^{\pi/4} \int_0^1 4\rho \cos\phi \, \rho^2 \sin\phi \, d\rho d\phi d\theta.$$