19. $\frac{d^2x}{dt^2} + k^2x = 0$, k real; when $t = 0$, $x = 0$, $\frac{dx}{dt} = v_0$. Verify your result completely.

ANS. $x = (v_0/k)\sin kt$.

20. $(D^3 + D^2 + 4D + 4)y = 0$; when $x = 0$, $y = 0$, $y' = -1$, $y'' = 5$.

ANS. $y = e^{-x} - \cos 2x$.

21. $\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + k^2x = 0$, $k > b > 0$; when $t = 0$, $x = 0$, $\frac{dx}{dt} = v_0$.

ANS. $x = (v_0/a)\,e^{-bt}\sin at$; where $a = \sqrt{k^2 - b^2}$.

Miscellaneous Exercises

Obtain the general solution unless otherwise instructed.

1. $(D^2 + 3D)y = 0$. ANS. $y = c_1 + c_2e^{-3x}$.

2. $(9D^4 + 6D^3 + D^2)y = 0$. ANS. $y = c_1 + c_2x + (c_3 + c_4x)\exp(-\frac{1}{3}x)$.

3. $(D^2 + D - 6)y = 0$. ANS. $y = c_1e^{2x} + c_2e^{-3x}$.

4. $(D^3 + 2D^2 + D + 2)y = 0$. ANS. $y = c_1e^{-2x} + c_2\cos x + c_3\sin x$.

5. $(D^3 - 3D^2 + 4)y = 0$. ANS. $y = c_1e^{-x} + e^{2x}(c_2 + c_3x)$.

6. $(D^3 - 2D^2 - 3D)y = 0$. ANS. $y = c_1 + c_2e^{3x} + c_3e^{-x}$.

7. $(4D^3 - 3D + 1)y = 0$. ANS. $y = c_1e^{-x} + (c_2 + c_3x)\exp(\frac{1}{2}x)$.

8. $(D^3 + 3D^2 - 4D - 12)y = 0$. ANS. $y = c_1\cosh 2x + c_2\sinh 2x + c_3e^{-3x}$.

9. $(D^3 + 3D^2 + 3D + 1)y = 0$. ANS. $y = e^{-x}(c_1 + c_2x + c_3x^2)$.

10. $(4D^3 - 21D - 10)y = 0$. ANS. $y = c_1e^{-2x} + c_2\exp(\frac{5}{2}x) + c_3\exp(-\frac{1}{2}x)$.

11. $(4D^3 - 7D + 3)y = 0$. ANS. $y = c_1e^{x} + c_2\exp(\frac{1}{2}x) + c_3\exp(-\frac{3}{2}x)$.

12. $(D^2 - D - 6)y = 0$; when $x = 0$, $y = 2$, $y' = 1$. ANS. $y = e^{3x} + e^{-2x}$.

13. $(D^4 + 6D^3 + 9D^2)y = 0$; when $x = 0$, $y = 0$, $y' = 0$, $y'' = 6$, and as $x \to \infty$, $y' \to 1$. For this particular solution, find the value of y when $x = 1$. ANS. $y = 1 - e^{-3}$.

14. $(D^3 + 6D^2 + 12D + 8)y = 0$; when $x = 0$, $y = 1$, $y' = -2$, $y'' = 2$.

ANS. $y = e^{-2x}(1 - x^2)$.

15. $(D^3 - 14D + 8)y = 0$.

ANS. $y = c_1e^{-4x} + c_2\exp[(2 + \sqrt{2})x] + c_3\exp[(2 - \sqrt{2})x]$.

16. $(8D^3 - 4D^2 - 2D + 1)y = 0$. ANS. $y = (c_1 + c_2x)\exp(\frac{1}{2}x) + c_3\exp(-\frac{1}{2}x)$.

17. $(D^4 + D^3 - 4D^2 - 4D)y = 0$.

18. $(D^4 - 2D^3 + 5D^2 - 8D + 4)y = 0$.

ANS. $y = e^x(c_1 + c_2x) + c_3\cos 2x + c_4\sin 2x$.

19. $(D^4 + 2D^2 + 1)y = 0$.

20. $(D^4 + 5D^2 + 4)y = 0$. ANS. $y = c_1\cos x + c_2\sin x + c_3\cos 2x + c_4\sin 2x$.

21. $(D^4 + 3D^3 - 4D)y = 0$.

22. $(D^5 + D^4 - 9D^3 - 13D^2 + 8D + 12)y = 0$.

ANS. $y = c_1e^x + c_2e^{3x} + c_3e^{-x} + e^{-2x}(c_4 + c_5x)$.

23. $(D^4 - 11D^3 + 36D^2 - 16D - 64)y = 0$.

24. $(D^2 + 2D + 5)y = 0$. ANS. $y = e^{-x}(c_1\cos 2x + c_2\sin 2x)$.

25. $(D^4 + 4D^3 + 2D^2 - 8D - 8)y = 0$.

26. $(4D^4 - 24D^3 + 35D^2 + 6D - 9)y = 0$.

ANS. $y = e^{3x}(c_1 + c_2x) + c_3\cosh\frac{1}{2}x + c_4\sinh\frac{1}{2}x$.

27. $(4D^4 + 20D^3 + 35D^2 + 25D + 6)y = 0$.

28. $(D^4 - 7D^3 + 11D^2 + 5D - 14)y = 0$.

29. $(D^3 + 5D^2 + 7D + 3)y = 0$.

30. $(D^3 - 2D^2 + D - 2)y = 0$. ANS. $y = c_1e^{2x} + c_2\cos x + c_3\sin x$.

31. $(D^3 - D^2 + D - 1)y = 0$.

32. $(D^3 + 4D^2 + 5D)y = 0$.

33. $(D^4 - 13D^2 + 36)y = 0$.

34. $(D^4 - 5D^3 + 5D^2 + 5D - 6)y = 0$.

ANS. $y = c_1\cosh x + c_2\sinh x + c_3e^{2x} + c_4e^{3x}$.

35. $(4D^3 + 8D^2 - 11D + 3)y = 0$.

36. $(D^3 + D^2 - 16D - 16)y = 0$.

37. $(D^4 - D^3 - 3D^2 + D + 2)y = 0$. ANS. $y = c_1e^x + c_2e^{2x} + e^{-x}(c_3 + c_4x)$.

38. $(D^3 - 2D^2 - 3D + 10)y = 0$.

39. $(D^5 + D^4 - 6D^3)y = 0$.

40. $(4D^3 + 28D^2 + 61D + 37)y = 0$. ANS. $y = c_1e^{-x} + e^{-3x}(c_2\cos\frac{1}{2}x + c_3\sin\frac{1}{2}x)$.

41. $(4D^3 + 12D^2 + 13D + 10)y = 0$.

42. $(18D^3 - 33D^2 + 20D - 4)y = 0$.

43. $(D^5 - 2D^3 - 2D^2 - 3D - 2)y = 0$.

ANS. $y = e^{-x}(c_1 + c_2x) + c_3e^{2x} + c_4\cos x + c_5\sin x$.

44. $(D^4 - 2D^3 + 2D^2 - 2D + 1)y = 0$.

45. $(4D^5 + 4D^4 - 9D^3 - 11D^2 + D + 3)y = 0$.

46. $(D^5 - 15D^3 + 10D^2 + 60D - 72)y = 0$.

47. $(D^4 + 2D^3 - 6D^2 - 16D - 8)y = 0$.

ANS. $y = e^{-2x}(c_1 + c_2x) + e^x(c_3\cos\sqrt{3}x + c_4\sin\sqrt{3}x)$.