Review Final **Name**
Math 5339 **Id**

Read each question carefully. Avoid simple mistakes. **You must show your work in order to get full credits**.

(1) Solve the following using characteristics method or substitution method

$$(a) \quad 2u_y - u_x = u$$
$$(b) \quad u_x + 5u_y = 6x$$

(2) Ex. Let Ω be the first quadrant in $\mathbb{R}^2$. Solve using characteristics method

$$xu_y - yu_x = u,$$
$$u(0, x) = f(x)$$

(3) [Evans Oct.29] Here is a general theorem on solving 1st-order PDE using characteristics.

Theorem. Let $u \in C^2(\Omega)$ solve the nonlinear first order PDE

$$F(Du, u, x) = 0 \quad x \in \Omega$$

Suppose $\mathbf{x}$ solves $x'(s) = D_pF(p(s), z(s), x(s))$. where $\mathbf{p}(s) = Du(x(s))$, and $z(s) = u(x(s))$. Then

$$p'(s) = -D_xF(p(s), z(s), x(s)) - D_zF(p(s), z(s), x(s))p(s)$$
$$z'(s) = D_pF(p(s), z(s), x(s)) \cdot p(s).$$

(4) Ex. $F(p(s), z(s), x(s)) = \mathbf{b}(x) \cdot Du(x) + c(x)u(x) = 0$ see [Evans, Oct29]

(5) Ex. [Evans, Oct] Use energy method to prove strong maximum principle for the heat equation.

(6) In Review Test 3, we have the uncertainty principle (UC). Prove a sharper version:

$$\int_{\mathbb{R}^n} |u|^2 \leq \frac{2}{n} \|\nabla u\|_2 \|xu\|_2 .$$

(7) (Bi-harmoinc functions) Solve $\Delta\Delta u = 0$ in $\mathbb{R}^d$, $d = 2, 3$.

(8) Apply Fourier series to solve periodic heat equation on the interval $\mathbb{T} = [0, 2\pi]$

$$u_t = ku_{xx} \quad x \in \mathbb{T}$$
$$u(0, x) = f(x)$$

(9) Solve the Schrödinger equation with periodic data on the torus $\mathbb{T}$

$$iu_t = -\frac{1}{2}u_{xx} \quad x \in \mathbb{T}$$
$$u(0, x) = f(x)$$

(10) (Sine-Gordon equation) In 1939 Frenkel and Kontorova introduced a problem arising in solid state physics to model the relationship between dislocation dynamics and plastic deformation of a crystal (Frenkel & Kontorova, 1939). From this study, an equation describing dislocation motion is

$$u_{xx} - u_{tt} = \sin u$$

here $u(t, x)$ is atomic displacement in the x-direction and the sin function represents periodicity of the crystal lattice. A traveling wave corresponding to the propagation of a dislocation is given by $u(t, x) = g(x - \xi t) = 4 \tanh(\exp(\frac{x-\xi t}{\sqrt{1-\xi^2}}))$.

(11) There are G'/G-method that deals with soliton waves for more general class of PDE, see [Lecture Notes]

(12) [NLS system, coupled BEC]

Deng-Shan Wang, Yu-Ren Shi, Kwok Wing Chow, Zhao-Xian Yu, Xiang-Gui Li, Matter-wave solitons in a spin-1 Bose-Einstein condensate with time-modulated external potential and scattering lengths. The European Physical Journal D, November 2013, 67:242, Date: 22 Nov 2013

Abstract.

In this paper, we present many matter-wave solitons in a system of three component Gross-Pitaevskii equation arising from the context of spinor Bose-Einstein condensates with time-modulated external potential and scattering lengths. The three component Gross-Pitaevskii equation with time-dependent parameters is first transformed into a three coupled nonlinear Schrdinger equation, then the exact soliton solutions of the three coupled nonlinear Schrdinger equation are given explicitly. Finally, the dynamics of the matter-wave solitons in the $F = 1$ spinor Bose-Einstein condensates is examined by specially choosing the frequency of the external potential. It is shown that when the frequency of the external potential is constant, there exist different kinds of matter-wave solitons as the atomic s-wave scattering lengths are varied about time, such as solitons with shape changing interactions, two-soliton bound states, squeezed matter-wave solitons, single bright and dark solitons. When the frequency of the external potential is time-modulated, there also exist various matter-wave solitons in the $F = 1$ spinor Bose-Einstein condensates, and we show that the time evolutions of the matter-wave solitons are sharply changed by the time-dependent trap frequency and nonlinear coefficients.

(13) T. Kanna, , R. Babu Mareeswaran, K. Sakkaravarthi, Non-autonomous bright matter wave solitons in spinor Bose-Einstein condensates. Physics Letters A Volume 378, Issue 3, 10 January 2014, Pages 158–170.

Abstract We investigate the dynamics of bright matter wave solitons in spin-1 Bose-Einstein condensates with time modulated nonlinearities. We obtain soliton solutions of an integrable autonomous three-coupled Gross-Pitaevskii (3-GP) equations using Hirota's method involving a non-standard bilinearization. The similarity transformations are developed to construct the soliton solutions of non-autonomous 3-GP system. The non-autonomous solitons admit different density profiles. An interesting phenomenon of soliton compression is identified for kink-like nonlinearity coefficient with Hermite-Gaussian-like potential strength. Our study shows that these non-autonomous solitons undergo non-trivial collisions involving condensate switching.

Keywords Spinor Bose-Einstein condensate; Three-coupled GrossPitaevskii equation; Similarity transformation; Hirota's bilinearization method; Bright soliton solution; Soliton interaction

Richard S. Tasgal and Y. B. Band, Sound waves and modulational instabilities in spinor Bose-Einstein condensates. arXiv.cond-mat.quant-gas, aug20, 2014

(14) Liu, W. M., Chui, S. T., Integrable models in Bose-Einstein condensates. (English summary) Recent developments in integrable systems and Riemann-Hilbert problems (Birmingham, AL, 2000), 59–90, Contemp. Math., 326, Amer. Math. Soc., Providence, RI, 2003.

Summary: "The theory of Bose-Einstein condensation of dilute gases in magnetic traps and optical lattices is reviewed from the point of view of the integrability of the equation of motion. The mean field theory–the time-dependent nonlinear Schrdinger equation (or the Gross-Pitaevskii equation)–provides a framework to understand the main features of the Bose-Einstein condensates and the role of the interaction between atoms. Using the inverse scattering method, we obtain the exact solutions of the one-dimensional single or coupled nonlinear Schrodinger equations, and explain the coherent properties, the collective excitations and the striation pattern of Bose-Einstein condensates reported in recent experiments. Using the periodic instanton method, we investigate the quantum tunneling of Bose-Einstein condensates in optical lattices under gravity in the 'Wannier-Stark localization' regime and 'Landau-Zener tunneling' regime. Our results agree with experimental data."

‡exact bring and dark soliton solutions for CNLS; nonlinear excitations of two-component BEC. Consider BEC with weakly interaction for the atomic gases, where soliton and vortices are featured as macroscopically excited Bose condensed states that are key phenomenon for superfluid. Bright and dark solitons in BEC correspond to the attractive and repulsive interactions (s-wave scattering length $a < 0$ or $a > 0$ reps.)

The motion or dynamics of 2-BEC is described by wave functions (ψ, φ) of CNLS

$$i\hbar\psi_t = -\frac{\hbar^2}{2m}\psi_{xx} + \frac{4\pi\hbar^2}{m}(a_{11}|\psi|^2 + a_{12}|\phi|^2)\psi \tag{1}$$

$$i\hbar\phi_t = -\frac{\hbar^2}{2m}\phi_{xx} + \frac{4\pi\hbar^2}{m}(a_{22}|\phi|^2 + a_{12}|\psi|^2)\phi \tag{2}$$

[1]

[1] a_{ij} are s-wave scattering lengths. If $i = j$, a_{ii} is for the length between same species. If otherwise $i \neq j$, a_{ij} is for different component

Consider the case of interest when $a_{ij} = a$ and where we can find the "Lax pair" in order to apply the inverse scattering method. Liu and Chui showed that

A. If $a < 0$, the exact bright soliton of BEC with attractive interaction the one soliton (1-soliton)

$$\begin{aligned}\psi(t,x) =& 2\eta_1\epsilon_1 \operatorname{sech}\left(2\eta_1\sqrt{4\pi a}[x - x_0 + 2t\xi_1\frac{\hbar}{m}\sqrt{4\pi a}]\right)\\ &\times \exp\left(-i[2\xi_1\sqrt{4\pi a}(x-x_0) + 8\pi\frac{\hbar}{m}a(\xi_1^2 - \eta_1^2)t]\right)\\ \phi(t,x) =& 2\eta_1\epsilon_2 \operatorname{sech}\left(2\eta_1\sqrt{4\pi a}[x - x_0 + 2t\xi_1\frac{\hbar}{m}\sqrt{4\pi a}]\right)\\ &\times \exp\left(-i[2\xi_1\sqrt{4\pi a}(x-x_0) + 8\pi\frac{\hbar}{m}a(\xi_1^2 - \eta_1^2)t])\right)\end{aligned}$$

here x_0 is the center position of the 1-soliton, the velocity is $v_1 = 2\xi_1\hbar/m\sqrt{4\pi a}$, ξ, η are real constants and in the construction of ϕ, ψ, λ is called the eigenvalue of this bright soliton.

Thus the density for the BEC are

$$\begin{aligned}|\psi(t,x)|^2 =& 4\eta_1^2\epsilon_1^2 \operatorname{sech}^2\left(2\eta_1\sqrt{4\pi a}[x - x_0 + 2t\xi_1\frac{\hbar}{m}\sqrt{4\pi a}]\right)\\ |\phi(t,x)|^2 =& 2\eta_1^2\epsilon_2^2 \operatorname{sech}^2\left(2\eta_1\sqrt{4\pi a}[x - x_0 + 2t\xi_1\frac{\hbar}{m}\sqrt{4\pi a}]\right)\end{aligned}$$

B. If $a > 0$ the exact dark soliton of BEC with repulsive interaction: for the Manakov case $a_{11} = a_{12} = a_{22}$, using Lax pair and inverse scattering transform the one-soliton are obtained

$$\begin{aligned}\psi(t,x) &= \epsilon_1\frac{(\xi_1 + i\eta_1)^2 + e^{2\gamma_1}}{1 + e^{2\gamma_1}}\\ \phi(t,x) &= \epsilon_2\frac{(\xi_1 + i\eta_1)^2 + e^{2\gamma_1}}{1 + e^{2\gamma_1}}\end{aligned}$$

where $\gamma_1 = \eta_1\sqrt{4\pi a}[x - x_0 + v_1 t]$, $v_1 = 2\xi_1\hbar/m\sqrt{4\pi a}$, in the IST λ called the eigenvalue of this dark soliton, the constants $\xi_1, \eta_1 \in \mathbb{R}$ satisfying $\xi_1^2 + \eta_1^2 = 1$. The density distribution of the BEC are then

$$\begin{aligned}|\psi(t,x)|^2 &= |\epsilon_1|^2(1 - \eta_1^2 \operatorname{sech}^2(\gamma_1))\\ |\phi(t,x)|^2 &= |\epsilon_2|^2(1 - \eta_1^2 \operatorname{sech}^2(\gamma_1))\end{aligned}$$

here η_1^2 is called the darkness of the soliton.

Exact periodic solution for striation pattern of 2-component BEC Let ψ, ϕ be the macroscopic wave functions of the different BEC species. Let a_{ij} denote the 1-dimensional reduced scattering lengths between the atoms for the same species

$(i = j)$ and for different species $(i \neq j)$. For cigar-shaped traps one can reduce the 3d to 1d problem where z is the axial direction of the external magnetic field. From experiment, the width of the magnetic domain is much smaller than the size of condensates. Hence the free CNLS without trap may capture the main feature of the experimental observations.

Consider the CNLS describing the BEC in cigar-shaped traps, where the Bose condensates are strongly confined along the z-axis and we can treat the system as a quasi-1d system. Liu and Chui found the Lax pair for the CNLS, then derive the exact solutions applying inverse scattering method.

$$i\hbar\psi_t = -\frac{\hbar^2}{2m}\psi_{zz} + \frac{4\pi\hbar^2}{m}(a_{11}|\psi|^2 + a_{12}|\phi|^2)\psi \tag{3}$$

$$i\hbar\phi_t = -\frac{\hbar^2}{2m}\phi_{zz} + \frac{4\pi\hbar^2}{m}(a_{22}|\phi|^2 + a_{12}|\psi|^2)\phi \tag{4}$$

One class of solutions exhibit a periodic structure in the density difference of the 2-component. Their result makes explicit dependence of the period of the density modulation on the energy of the initial state, which may be tested experimentally. It also suggests that there is a range of compositions at which the mixture exhibits the metastable intermediate time periodic state and that this range is different for the ^{87}Rb and the ^{23}Na systems.

In [Liu and Chui] first find a "Lax pair" whose compatibility conditions reproduce (4), (3). Then transfer these two equations or system into solving a set of inverse scattering equations that determine the 3 by 3 scattering matrix from $z \to -\infty$ to $z \to \infty$. Physically one looks at the discrete spectrum.

IST allows to reduce the inverse scattering equations to a set of linear inhomogeneous algebraic equations. Constructing the Lax pair $L = Q_1, M = \lambda Q_1 + \frac{i}{2}Q_2$ where

$$Q_1 = \begin{pmatrix} -i\lambda & \phi_1 & \phi_2 \\ \phi_1 & i\lambda & 0 \\ \phi_2 & 0 & i\lambda \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} |\phi_1|^2 + |\phi_2|^2 & \partial\phi_1/\partial\tau & \partial\phi_2/\partial\tau \\ \partial\phi_1^*/\partial\tau & -|\phi_1|^2 & -\phi_1^*\phi_2 \\ \partial\phi_2^*/\partial\tau & -\phi_1\phi_2^* & -|\phi_2|^2 \end{pmatrix}$$

[2] where $\tau = 4\pi a\hbar t/m$, $r = \sqrt{4\pi a}x$ and $\lambda = \xi + i\eta$ denotes the spectral parameter. In light of the IST, we verify that the solution matrix

$$\Psi(\lambda, r \to -\infty) = \begin{pmatrix} e^{-i\lambda r} & 0 & 0 \\ 0 & e^{i\lambda r} & 0 \\ 0 & 0 & e^{i\lambda\tau} \end{pmatrix} \tag{5}$$

$$\Psi(\lambda, r \to \infty) = \begin{pmatrix} e^{-i\lambda r} & 0 & 0 \\ 0 & e^{i\lambda r} & 0 \\ 0 & 0 & e^{i\lambda\tau} \end{pmatrix} S^T \tag{6}$$

here $S = (c_{ij})_{33}$ the matrix of scattering coefficients c_{ij} and S the unitary scattering matrix satisfying $S^+S = I$ for real λ.

For $a_{11} = a_{22} = a_{12}$ as in Manakov Liu-Chui found the Lax pair above:

$$\Phi_r = L\Phi$$
$$\Phi_\tau = M\Phi$$

and, let $J = (+, -, -)$ be the diagonal matrix, then the x-evolution of S is given by

$$S_r = i\lambda^2[J, S].$$

[Liu-Chui] obtain the exact periodic solutions

$$\begin{aligned} \phi_1(t,z) =& c_1 \,\mathrm{sn}(\sqrt{8\pi a_{12}}\,\alpha(z - \sqrt{8\pi a_{12}}\,\hbar\beta t/m), k) \\ & \exp(i\sqrt{8\pi a_{12}}\,\beta(z - \sqrt{2\pi a_{12}}\,\hbar\omega_1 t/m\beta)) \\ \phi_2(t,z) =& c_2 \,\mathrm{cn}(\sqrt{8\pi a_{12}}\,\alpha(z - \sqrt{8\pi a_{12}}\,\hbar\beta t/m), k) \\ & \exp(i\sqrt{8\pi a_{12}}\,\beta(z - \sqrt{2\pi a_{12}}\,\hbar\omega_2 t/m\beta)) \end{aligned}$$

here sn, cn are the sine-amplitude and the cosine-amplitude Jacobian elliptic functions with modulus k; ω_i are the corresponding frequencies determined by the I.C. of the system.

(15) date back in 2003 Kanna and Lakshmanan already has exact solitons of CNLS of N-systems: it is in the folder soliton-CNLS03.pdf

also, in a companion paper 2001 [arXiv nlin.SI] Kanna and Lakshmanan "Exact soliton solutions, shape changing collisions and partially coherent soliton in CNLS" the bright one-soliton and two-soliton of the 3-CNLS was studied

$$iq_{jz} + q_{jtt} + 2\mu(|q_1|^2 + |q_2|^2 + |q_3|^2)q_j = 0 \qquad j = 1, 2, 3$$

to show that

$$\begin{aligned} (q_1, q_2, q_3)^T =& \frac{e^{\eta_1}}{1 + e^{\eta_1 + \eta_1^* + R}}(\alpha_1^{(1)}, \alpha_1^{(2)}, \alpha_1^{(3)})^T \\ =& \frac{k_{1R}e^{i\eta_{1I}}}{\cosh(\eta_{1R} + R/2)}(A_1, A_2, A_3)^T \end{aligned}$$

[2] one of the variables should be ∂r

here $\eta_1 = k_1(t + ik_1 z)$, $A_j = \alpha_1^{(j)}/\Delta$, $\Delta = (\mu \sim_{j=1}^{3} |\alpha_1^{(j)}|^2)^{1/2}$; $\alpha_1^{(j)}, k_1$ are four arbitrary complex parameters. Moreover $k_{1R}A_j$ is the amplitude of the jth mode and $2k_{1I}$ the soliton velocity.

(16) Guo-Quan Zhou and Nian-Ning Huang An N-soliton solution to the DNLS equation based on revised inverse scattering transform. Journal of Physics A: Mathematical and Theoretical Volume 40 Number 45. J. Phys. A: Math. Theor. 40 13607. doi:10.1088/1751-8113/40/45/008 Published 23 October 2007.

Abstract. Based on a revised version of inverse scattering transform for the derivative nonlinear Schrodinger (DNLS) equation with vanishing boundary condition (VBC), the explicit N-soliton solution has been derived by some algebra techniques of some special matrices and determinants, especially the BinetCauchy formula. The one- and two-soliton solutions have been given as the illustration of the general formula of the N-soliton solution. Moreover, the asymptotic behaviors of the N-soliton solution have been discussed.

(17) D Zhang, T Yan, H Cai, Explicit multisoliton solution to the coupled nonlinear Schrodinger equations. Wuhan University Journal of Natural Sciences, 2013 - Springer.

Abstract. Based on the inverse scattering transform for the coupled nonlinear Schrodinger (NLS) equations with vanishing boundary condition (VBC), the multisoliton solution has been derived by some determinant techniques of some special matrices and determinants ...

Qing Ding, The NLS- equation and its $SL(2,R)$ structure. Journal of Physics A: Mathematical and General Volume 33 Number 34

Qing Ding 2000 J. Phys. A: Math. Gen. 33 L325. doi:10.1088/0305-4470/33/34/101

Abstract. The relationship of the group $SL(2,R)$ and the NLS-equation is presented. As a consequence, the $SL(2,R)$ gauge equivalence between the NLS- and the $M-HF$ model is proved, which provides a new example in geometrically explaining dynamical properties of soliton equations by the $SL(2,R)$ structure.

(18) [dNLS] Consider the derivative NLS for $b \geq 0$

$$iu_t = -u_{xx} - i|u|^2 u_x - b|u|^4 u \qquad (t,x) \in \mathbb{R}^{1+1}$$

which came from nonlinear optics, plasma physics, etc. cf. [M. Ohta, arXiv.math.AP, 14]

Seek a traveling wave solution ansatz $u = e^{i\lambda t}\phi_\omega(x - \sigma t)$, $\omega = (\lambda, \sigma) \in \Omega := \{(\lambda,\sigma) \in \mathbb{R}^2 : \sigma^2 < 4\lambda\}$.

$$\phi_\omega(x) = \eta_\omega(x)\exp(\frac{i\sigma x}{2} - \frac{i}{4}\int_{-\infty}^{x} |\eta_\omega(y)|^2 dy)$$
$$\eta_\omega(x) = (\frac{2(4\lambda - \sigma^2)}{-\sigma + \sqrt{\sigma^2 + \gamma(4\lambda - \sigma^2)}\cosh(\sqrt{4\lambda - \sigma^2 x})})^{1/2}$$

where $\gamma = 1 + 16b/3$. The $\phi_\omega(x)$ and η_ω solve the elliptic (indecent of t)

$$-u_{xx} + \sigma i u_x - i|u|^2 u_x - b|u|^4 u = -\lambda u \qquad x \in \mathbb{R}$$
$$-\eta_{xx} + \frac{\sigma}{2}|u|^2 u - \frac{3\gamma}{16}|u|^4 u = -\frac{4\lambda - \sigma^2}{4}\eta \qquad x \in \mathbb{R}$$

The dNLS is generated by the Hamiltonian

$$iu_t = \frac{\delta H}{\delta u}$$

that is the gradient of energy is parallel to the rate of change, or "velocity" by a complex factor. Here $H : H^1(\mathbb{R}) \to \mathbb{R}$, where H^1 is endowed with real inner product $\langle u, v\rangle_{H^1} = \langle u, v\rangle_2 + \langle u_x, v_x\rangle_2$ and in $L^2(\mathbb{R})$

$$\langle u, v\rangle := \Re \int u\bar{v}dx.$$

$$H[u] = \frac{1}{2}\|u_x\|_2^2 - \frac{1}{4}\langle i|u|^2 u_x, u\rangle - \frac{b}{6}\|u\|_6^6$$
$$\frac{\partial H}{\partial u} = -u_{xx} - i|u|^2 u_x - b|u|^4 u \qquad in\ H^{-1}$$

(19) Show that the following are conserved
 (a) $M[u] = \frac{1}{2}\int |u|^2$
 (b) $p[u] = \frac{1}{2}\langle iu_x, u\rangle_2$
 (c) $E[u] = \frac{1}{2}\|u_x\|_2^2 - \frac{1}{4}\langle i|u|^2 u_x, u\rangle_6 - \frac{b}{6}\|u\|_6^6$

 Remark One can compute the exact value of the mass, momentum and energy for the soliton waves ϕ_ω. see [Ohta14]

(20) [Evans Nov.26] gives a similar solution for traveling waves for the reaction diffusion model

$$u_t = u_{xx} + f(u) \qquad R_+ \times \mathbb{R}$$

**Read the details therein. **See also [Strauss, 14.4] on bifurcation theory.

References.

Juan Belmonte-Beitia, Víctor M. Pérez-García and Pedro J. Torres, " Solitary waves for linearly coupled nonlinear Schrodinger equations with inhomogeneous coefficients".

Abstract Motivated by the study of matter waves in Bose-Einstein condensates and coupled nonlinear optical systems, we study a system of two coupled nonlinear Schrodinger equations with inhomogeneous parameters, including a linear coupling. For that system we prove the existence of two different kinds of homoclinic solutions to the origin describing solitary waves of physical relevance. We use a Krasnoselskii fixed point theorem together with a suitable compactness criterion. Key words: Nonlinear Schrodinger systems, solitary waves, fixed point theorems in cones.

To solve for the soliton there is applied a fixed point theorem referred in this article which is due to Krasnoselskii. [3]

Theorem 0.1. *Let X be a B space, and let $P \subset X$ be a cone in X. Assume Ω_1, Ω_2 are open subsets in X with $0 \in \Omega_1, \overline{\Omega}_1 \subset \Omega_2$ and let $T : P \cap (\overline{\Omega_2} \setminus \Omega_1) \to P$ be a completely continuous operator s.t. one of the following is satisfied*

(1) $\|Tu\| \leq \|u\|$ *if* $u \in P \cap \partial\Omega_1$ *and* $\|Tu\| \geq \|u\|$ *if* $u \in P \cap \partial\Omega_2$
(2) $\|Tu\| \geq \|u\|$ *if* $u \in P \cap \partial\Omega_1$ *and* $\|Tu\| \leq \|u\|$ *if* $u \in P \cap \partial\Omega_2$.

Then T has at least one fixed point in $P \cap (\overline{\Omega_2} \setminus \Omega_1)$.

A. Jungel and RADA-MARIA Weishaupl, BLOW-UP IN TWO-COMPONENT NONLINEAR Schrodinger SYSTEMS WITH AN EXTERNAL DRIVEN FIELD.

Abstract. A system of two nonlinear Schrodnger equations in up to three space dimensions is analyzed. The equations are coupled through cubic mean-field terms and a linear term which models an external driven field described by the Rabi frequency. The intraspecific mean-field expressions may be non-cubic. The system models, for instance, two components of a Bose-Einstein condensate in a harmonic trap. Sufficient conditions on the various model parameters for global-in-time existence of strong solutions are given. Furthermore, the finite-time blow-up of solutions is proved under suitable conditions on the parameters and in the presence of at least one focusing nonlinearity. Numerical simulations in one and two space dimensional equations verify and complement the theoretical results. It turns out that the Rabi frequency of the driven field may be used to control the mass transport and hence to influence the blow-up behavior of the system.

see 05-p11weishaeupl.pdf

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X. Carvajal,M Panthee, and M. Scialom, On the critical KdV equation with time-oscillating nonlinearity. Differential Integral Equations Volume 24, Number 5/6 (2011), 541–567.

Abstract. We investigate the initial-value problem (IVP) associated with the equation

$$u_t + \partial_x^3 u + g(\omega t)\partial_x(u^5) = 0,$$

where g is a periodic function. We prove that, for given initial data $\phi \in H^1(\mathbb{R})$, as $|\omega| \to \infty$, the solution u_ω converges to the solution U of the initial-value problem associated with

$$U_t + \partial_x^3 U + m(g)\partial_x(U^5) = 0,$$

with the same initial data, where $m(g)$ is the average of the periodic function g. Moreover, if the solution U is global and satisfies $\|U\|_{L_x^5 L_t^{10}} < \infty$, then we prove that the solution u_ω is also global provided $|\omega|$ is sufficiently large.

X Carvajal, P Gamboa, M Panthee, A system of coupled Schrödinger equations with time-oscillating nonlinearity. International Journal of Mathematics 23 (11). World Scientific Publishing Company

abstr. This paper is concerned with the initial value problem (IVP) associated to the coupled system of supercritical nonlinear Schrodinger equations

$$\begin{aligned} iu_t + \Delta_x u + \theta_1(\omega t)(|u|^{2p} + \beta|u|^{p-1}|v|^{p+1})u = 0 \\ iv_t + \Delta_x v + \theta_2(\omega t)(|v|^{2p} + \beta|v|^{p-1}|v|^{p+1})u = 0 \end{aligned}$$

where θ_1 and θ_2 are periodic functions, which has applications in many physical problems, especially in nonlinear optics. We prove that, for given initial data $\phi, \psi \in H^1(\mathbb{R}^n)$, as $|\omega| \to \infty$, the solution (u_ω, v_ω) of the above IVP converges to the solution (U, V) of the IVP associated to

$$\begin{aligned} iU_t + \Delta_x U + I(\theta_1)(|U|^{2p} + \beta|U|^{p-1}|V|^{p+1})U = 0 \\ iV_t + \Delta_x V + I(\theta_2)(|v|^{2p} + \beta|V|^{p-1}|U|^{p+1})V = 0 \end{aligned}$$

with the same initial data, where $I(g)$ is the average of the periodic function g. Moreover, if the solution (U, V) is global and bounded then we prove that the solution (u_ω, v_ω) is also global provided $|\omega| \gg 1$.

Ref. J. Chen and B. Guo, Blow-up profile to the solutions of two-coupled Schrödingier equation with harmonic potential. J. Math. Phys. 50 (2009), 023505.

abs. The model of the following two-coupled Schrodinger equations

$$iu_t + \frac{1}{2}\Delta u = (g_{11}|u|^{2p} + g|u|^{p-1}|v|^{p+1})u, \quad (t,x) \in \mathbb{R}_+ \times R^N, \tag{7}$$

$$iv_t + \frac{1}{2}\Delta v = (g|u|^{p+1}|v|^{p-1} + g_{22}|v|^{2p})v \tag{8}$$

is proposed in the study of the Bose-Einstein condensates [Mitchell, et al., "Self-traping of partially spatially incoherent light," Phys. Rev. Lett.77, 490 (1996)]. We prove that for suitable initial data and p the solution blows up exactly like δ function. As a by-product, we prove that similar phenomenon occurs for the critical two-coupled Schrodinger

equations with harmonic potential [Perez-Garcia, V. M. and Beitia, T. B., Sybiotic solitons in heteronuclear multicomponent Bose-Einstein condensates, Phys. Rev. A72, 033620 (2005)],

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