Project 2	**Name**
Math 5530	**Id**

Read each question carefully. Avoid simple mistakes. *You must show your work step by step to justify the answer in order to receive full credits.*

[The assignment covers sections 9.7, 9.8, 9.9, 11.1-11.4, 11.7. Question(s) marked with $*$ is for graduate students]

1. [8] The force in an electrostatic field given by $f(x, y, z)$ has the direction on the gradient. Find ∇f and its value at P where

$$f(x, y) = \frac{x}{x^2 + y^2}, \qquad P = (1, -2)$$

2. [8] In the study of the motion of a fluid in a given region Ω in the plane, the divergence div $\mathbf{u}$ measures outflow minus inflow, generally speaking. In the incompressible (constant density) case, if the flow has no sources or sinks in Ω, that is, div $u = 0$, then the velocity field $\mathbf{u}$ is referred to as **solenoidal**. For what u_3 is $\mathbf{u} = \langle e^x \cos y, e^x \sin y, u_3 \rangle$ solenoidal?

   Clue: div $\mathbf{u} = \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z}$.

3. [8] Let $\mathbf{u}$ be the velocity vector of a steady fluid flow.

   (a) Is the flow irrotational?

   (b) Incompressible?

   (c*) Find the streamlines (the paths of the particles)

$$\mathbf{u} = \langle -y, x, \pi \rangle.$$

   (Clues: (a) Compute curl $\mathbf{u} = 0$ or not;
   (b) Calculate div $\mathbf{u} = 0$ or not.
   (c) Read Section 9.9.

4. [8] Find the Fourier series Ff of the 2π-periodic function

$$f(x) = \begin{cases} 1 & x \in [0, \pi) \\ 0 & x \in [\pi, 2\pi). \end{cases}$$

where $Ff(x) := \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$, and for all $n = 0, 1, 2, \ldots$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)dx$$
$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin(nx)dx.$$

Compare Ff and f and evaluate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$.

5. [8] Represent $f(x) = \begin{cases} \sin x, & x \in (0,\pi) \\ 0, & x > \pi \end{cases}$ as an cosine-Fourier integral

$$f(x) = \int_0^{\infty} A(k)\cos(kx)dk,$$

where $A(k) = \frac{2}{\pi}\int f(x)\cos(kx)dx$.